DESIGN AND ANALYSIS OF HUMANITARIAN AND PUBLIC HEALTH LOGISTICS SYSTEMS

A Thesis
Presented to
The Academic Faculty

by

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In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy in the
H. Milton Stewart School of Industrial and Systems Engineering

Georgia Institute of Technology
December 2010

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ACKNOWLEDGMENTS

I would like to thank the members of my committee, Alan Erera, Özlem Ergun, Nathaniel Hupert, Pinar Keskinocak, Nicoleta Serban, and Julie Swann, for their time and effort. Each of them has contributed to my formation as a researcher and educator, and I am grateful for their support. Their own commitments to making a difference in the world through their work have been instructional and inspiring.

I am particularly grateful to my thesis advisors, Özlem Ergun and Julie Swann, for their guidance and support throughout this process. Before I applied to graduate school, I knew that I wanted to work in the area of humanitarian logistics. I sought to get the best training I could in operations research and logistics methodologies, while hoping to find an advisor sympathetic to my application interests. Instead, I have had the privilege of learning from and working with two people whose passion for health and humanitarian applications is remarkable. At each stage of the graduate school process, they have been mindful of the steps needed not only to move our research forward, but also to help me prepare for future stages of my career. Their flexibility has afforded me the ability to balance both my professional and personal priorities. I am thankful to count each of them as a mentor and a friend.

I would like to express my gratitude to my colleagues at Kansas State University. Their encouragement and guidance during my undergraduate studies played an important role in my decision to pursue the PhD, and they continue to serve as invaluable mentors. I am especially grateful to the faculty and staff in the Department of Industrial and Manufacturing Systems Engineering, who provided access to facilities for summer research. I look forward to joining them soon and working together to prepare K-State IE's for rewarding careers.

Much of the research presented in this thesis was supported by the National Science Foundation. Additional support was provided by the Georgia Tech Focused Research Program on Humanitarian Response, the Harold R. and Mary Anne Nash Junior Faculty Endowment Fund, the ISyE Morris Fellowship, Phi Kappa Phi, the Material Handling Education Foundation, Inc., the Institute of Industrial Engineers Gilbreth Memorial Fellowship, the Google Anita Borg Memorial Scholarship, the Council of Supply Chain Management Professionals Atlanta Roundtable, and the Georgia National Defense Transportation Association. I deeply appreciate the support of these organizations and benefactors. I also gratefully acknowledge the Waffle House Restaurants personnel for sharing generously of their time and experience in the preparation of the work in Chapter 4.

Many thanks to my fellow ISyE PhD students, too many to mention by name, who have challenged and supported me during this journey. I appreciate the ultimate frisbee games, coffee breaks, study sessions, and advice they have shared. I treasure their friendship and look forward to celebrating many more professional and personal milestones to come.

To my "Atlanta family", I cannot express the depth of my gratitude. Who knew that potluck neighbors in the Graduate Living Center would become lifelong friends? They made the hard days bearable and turned a new city into a new home. Thanksgiving and Lunar New Year will always bring fond memories of our many celebrations and the abundance of "interesting" foods. Between sharing traditions from our respective cultures and creating traditions of our own, my life is richer because I shared this graduate school journey with them.

My family has been an unwavering source of strength and support. To the Heier, Cox, Stamm, and Chopp families, I express my sincere thanks for their prayers, understanding during the toughest days, and companionship always. I am especially grateful to my parents, Dave and Lee Ann, whose love and confidence in my ability to achieve my goals have given me both roots and wings. Russell, A.J., Bridget, Joel, and Caleb remind me of the most important things in life. My deepest appreciation is for my husband, Mike, without whose patience, encouragement, sacrifice, and confidence this thesis would not have been written. Thanks for sharing in my life's journey.

Numerous friends and mentors not listed by name have also supported me in this effort. Please know that I am truly grateful.

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SUMMARY

This thesis considers the design and analysis of humanitarian supply chains, by which we mean those systems that deliver goods and services in response to natural or man-made disasters as well as ongoing public health challenges. In the first part of the thesis, we introduce a class of problems motivated by humanitarian logistics systems with decentralized decision makers. In contrast to traditional optimization problems in which a centralized planner determines the actions of all entities in the system, decentralized systems are characterized by individual decision makers who make choices to optimize their own objectives and whose actions impact the overall system performance. Decentralized systems often perform poorly in comparison to centralized ones, but centralization is costly or impractical to implement in many circumstances. The goal of this part of the thesis is to characterize the impact of decentralized decision making and identify ways to mitigate this impact. Using concepts from optimization and game theory, we model systems in which individuals choose a facility to visit to receive service, such as during a disaster response, making their choices based on travel time, congestion, and weights on congestion. These weights represent the relative importance individuals place on congestion in their objectives. We provide an efficient algorithm for finding a stable, or equilibrium, solution from which no individual can improve her own objective value by switching unilaterally. We show that the worst- and best-case performances of decentralized solutions depend on the importance individuals place on congestion. Finally, we introduce a mechanism that, if implemented, ensures that any chosen solution is also a stable decentralized solution; the chosen solution could be the optimal solution found by a centralized planner, for example. The mechanism acts by influencing the importance individuals

place on congestion, and we characterize the values that this importance *can* and *must* be to achieve stability. We introduce models to find values of the mechanism that optimize particular policy objectives and show that these models can be solved efficiently.

The second part of the thesis describes the application of the ideas developed in the first part to data from a large-scale effort to deliver a limited supply of products to a large number of people in a short time. The goal of this part of the thesis is to understand the impact of decentralized decision making on local access to an actual product and quantify correlations between inequities in access and socioeconomic variables. We find that both the centralized and decentralized systems lead to inequity in access, but the impact is greater in decentralized systems with user choice. The differences in access are correlated with several socioeconomic variables, including income and racial/ethnic minority group, but these relationships vary across geographic space. This study integrates tools from optimization, game theory, spatial statistics, and geographic information systems in a novel way. The results confirm the importance of accounting for decentralized behavior in system design and point to opportunities to use the mechanism from the first part of the thesis in future distribution efforts of this nature. The study also leads to policy recommendations, namely that planners consider the impact on equity prior to implementing distribution plans and work to recruit additional service providers in areas that have exhibited inequities in the past.

The third part of the thesis employs empirical methods to characterize a successful humanitarian supply chain and identify practices from which other organizations can learn to improve their operations. The hurricane response process used by Waffle House Restaurants has been recognized nationally for its effectiveness. We document the process and describe the supply chain concepts that contribute to its success. Further, we place the company's practices in the context of the literature on supply chain disruption, crisis management, and humanitarian logistics. This study provides insight for other organizations that seek to improve their resilience to supply chain disruptions, whether these are caused by natural disasters or other events. The study also led to the creation of teaching materials to help business and engineering students identify the challenges faced in humanitarian supply chains, the ways that operations research methodologies can be used to improve decisions, and the opportunities for cross-learning between humanitarian organizations and the private sector.

CHAPTER I

INTRODUCTION

Disasters and public health challenges continue to have significant impact on the lives and livelihoods of people worldwide. For example, in 2009 alone more than 119 million people were affected by disasters which caused over US$ 41.3 billion in damages [99]. The same year, the international public health community faced a global influenza pandemic. Thus far in 2010, we have witnessed a devastating earthquake in Haiti, monumental flooding in Pakistan, and a host of other events that, while perhaps less visible, are still challenging for the international community. Preparedness for, response to, and mitigation of these humanitarian crises can be improved using quantitative logistics methodologies. Although some of the earliest applications of operations research and management science were in the public sector, humanitarian supply chain and logistics problems have received much less attention from the profession than their private sector counterparts. Many of these problems require the discovery of novel approaches, because the constraints and objectives differ from seemingly similar problems arising in the private sector. Solving these problems advances our understanding of theory as well as practice.

In both private sector and humanitarian supply chains, decision makers face fundamental questions about facility location, transportation network design, forecasting, inventory management, routing, scheduling, and allocation of resources to meet demand for goods and services. Additional challenges that are particularly present in humanitarian supply chains include high uncertainty in both supply and demand, the need for a quick and dynamic response, limited or damaged infrastructure, and a wide variety of agencies interacting while trying to pursue their own objectives.

Together, these characteristics often lead to *decentralized decision making.* System users are said to be decentralized because each individual or agency acts to optimize his own objective, but these decisions impact other users and the performance of the overall system. This is in contrast to the classical centralized optimization framework in which a single decision maker determines the actions within the system based on a global objective. Decentralization arises in humanitarian supply chains, for example, when limited infrastructure prohibits system-wide communication, urgency takes priority over time spent on coordination between decision makers, or decision makers have differing objectives. The performance of a decentralized system can be much worse than that of a system in which a central decision maker dictates choices, but centralized control is unrealistic or very costly in many practical cases.

This thesis considers the design and analysis of humanitarian supply chains, by which we mean those systems that deliver goods and services in response to natural or man-made disasters as well as ongoing public health challenges. The contributions of this thesis are threefold. First, we introduce a class of problems motivated by humanitarian logistics systems with decentralized decision makers and characterize theoretical properties of this class. Second, we demonstrate the applicability of the framework developed in the first part using data from a large-scale response effort. We illustrate how, when integrated with spatial statistics and geographic information systems, these methods can help assess inequities in distribution efforts. Finally, we adopt empirical methods to characterize a successful humanitarian supply chain implemented by a private sector company and identify practices from which other organizations can learn to improve their operations.

Both the first and second parts of the thesis are concerned with decentralized decision making in humanitarian supply chains. The problems we study are motivated by humanitarian response scenarios in which individuals choose, from a number of distribution sites, the location that they will visit to obtain needed supplies. The contributions of the first part, described in Chapter 2, are as follows:

- We introduce the Facility-Specific Congestion Weights Problem (FSCWP), in which individuals choose to visit one facility among a set of facilities opened by a centralized planner. We consider individual objectives that include travel time, congestion, and weights on congestion. Here, congestion depends on the number of other individuals at the facility and weights are used to represent the importance that individual decision makers place on congestion in their objectives.

- We provide a polynomial time algorithm for finding an equilibrium solution, or one from which no individual can deviate and improve her own solution, regardless of the values of congestion weights.

- We provide new bounds on the prices of anarchy and stability (respectively, worst- and best-case performance of the decentralized system in comparison to a centralized one) for the FSCWP. We find that the bounds depend on the congestion weight values allowed in the individuals' objective functions.

- We introduce the concept of *equilibrium-obtaining* congestion weight vectors, defined as those congestion values for which the centralized optimal solution is also an equilibrium solution. We demonstrate that it is always possible to find such a vector, and characterize the set of such vectors by examining what their values *can* and *must* be. We find that any given component of the congestion weight vector can be arbitrarily large and that the minimum value it must assume corresponds to the objective value of a particular shortest path problem. The method used to find a congestion weight vector for which the centralized solution is an equilibrium can also be used to find such a vector for any chosen solution.

- We develop optimization models that can be solved efficiently to find equilibrium-obtaining congestion weight vectors that optimize particular policy objectives.

The second part of the thesis describes the adaptation and application of ideas from Chapter 2 to a large-scale effort to distribute a product in a limited amount of time. The principle contributions of the second part are described in Chapter 3.

- We adapt the models from Chapter 2 to include facility capacity, defined as the quantity of product available, in both the centralized and decentralized objectives.

- We demonstrate that the methods introduced in Chapter 2 are applicable to large-scale problems arising in practice. We solve the decentralized model with approximately 80,000 communities of 100 people each and over 2,000 facilities with total stock of more than 2,000,000 products and compare the results to those of a centralized model of the same system.

- We quantify individuals' access to the product as a function of distance, congestion, and facility capacity and identify socioeconomic variables, including income, racial/ethnic minority groups, population density, and availability of service providers, that are associated with inequities in access.

- We integrate tools from geographic information systems (GIS), optimization, game theory, and spatial statistics. This novel integration makes use of data from many sources to explain the impact of past decisions about distribution system design, which in turn can lead to improved decisions and policies in the future.

The final part describes the hurricane response supply chain of Waffle House Restaurants. Chapter 4 contains the following contributions:

- We describe Waffle House Restaurants' successful efforts to prepare for and respond to hurricanes and the role of supply chain management principles, including supply and demand management and inventory planning, in this process.

- We relate the company's success to theoretical concepts from the literature on crisis management, supply chain disruption, and humanitarian logistics.

- We provide lessons for other companies in the private sector as well as government and humanitarian agencies that wish to improve their readiness for disasters or other supply chain disruptions.

- We use the information from this study to develop teaching materials that emphasize the opportunities for improving decisions with operations research methodologies and for cross-learning between public and private sector logistics operations. These materials have been used in undergraduate, graduate, and professional education settings with humanitarian practitioners as well as business and engineering students.

Together, the contributions of this thesis advance the understanding of humanitarian and public health logistics systems and provide insights for improving their performance. This work also has implications for other systems where user choice plays a role, including the health care sector and retail systems that distribute high-demand, low-supply items such as popular novels or electronics. The integration of spatial statistics, optimization, game theory, and GIS can lead to insights regarding the design and assessment of many systems where equity is important, such as the availability of housing, food retailers, or financial services. In Chapter 5, we describe future research motivated by the thesis.

CHAPTER II

INCORPORATING BEHAVIOR TO IMPROVE THE PERFORMANCE OF DECENTRALIZED NETWORKS

2.1 Introduction

We consider networks characterized by decentralized decision making. Classical optimization frameworks assume the existence of a centralized planner who determines all the actions within the system to optimize a system-wide objective, but many examples of decentralized systems exist in practice. In such systems, individuals make decisions to optimize local objectives, but the choices of decentralized users impact the entire system. Decentralized systems can perform poorly in comparison to their centralized counterparts. However, centralization is often impractical or costly to implement. In this chapter, we describe a class of decentralized network problems, quantify the impact of decentralization, and develop ways to improve system performance.

In the decentralized problems we study, individuals choose to visit one of a number of existing service facilities. The decentralized decision makers seek to minimize their own costs of receiving service, but the centralized objective is to minimize the total cost of providing service to all individuals, which is measured by the total travel time and congestion experienced by all individuals. This problem arises in a number of application areas in which service rates depend on individuals' choices of routes or service facilities, including networks of retail outlets or traffic networks. It is also common in public health and humanitarian operations. For example, when agencies open facilities to distribute goods or relief items in an emergency, residents choose the facility they will visit to obtain needed items. A similar circumstance arises in the allocation and use of medicine and supplies during an influenza pandemic. In many countries in Africa, individuals seeking antiretroviral treatments for HIV choose which clinic to visit from among those dispensing treatments. The common element in each of these and other decentralized network systems is that individuals make choices based on their own objectives, but these choices impact others and the overall system performance.

Modeling individual decision behavior is a key component of this research. When individuals choose a facility to visit, travel time and congestion or wait time at the facility are important components. The weight, or importance, placed on congestion also plays a critical role in individuals' evaluations of alternative facilities. In the problems we consider, the congestion weights can differ across facilities. Individuals may perceive regional health facilities differently than local clinics, for example, and value their wait at each site differently. Differing levels of information may be available regarding facility status as a result of media or social communication tools, also impacting the importance individuals place on congestion at the facilities. In this chapter, we introduce the Facility-Specific Congestion Weights Problem to model decentralized objectives that include travel time, congestion, and facility-specific weights on congestion arising in such scenarios.

In addition to introducing the decentralized problem from the individuals' perspectives, we define a central planner's problem to serve as a benchmark for the decentralized system. The planner's problem, as in traditional optimization, assumes that a single entity controls all the actions within the system in a way that minimizes total travel time and congestion. It is used as a basis for comparison to determine the additional cost that is incurred when decisions within the system are made by decentralized users rather than a single centralized planner. Converting to a centralized system is not possible in many contexts, but understanding the centralized problem can help in the design of better decentralized systems and mechanisms for improving performance even in circumstances where central control is impractical.

This research advances the understanding of decentralization in networks where individuals choose among facilities to receive services and makes three primary contributions. We first quantify the impact of decentralized decision making on system performance and show that the objective individuals use when making decisions has significant impact on network performance. Secondly, we demonstrate that systems that are designed from a perspective that assumes centralized control are subject to poor performance when they really operate in a decentralized way. However, our final contribution is the introduction of approaches for mitigating this impact in systems where centralization is not possible. To do so, we integrate tools from optimization and game theory to develop mathematical models that capture individual decision behavior and present algorithms for solving these models. We demonstrate that mechanisms that account for behavior can improve decentralized system performance. Ultimately, these results can be used to inform strategic and operational decisions made by those who manage decentralized systems. They help answer such questions as how to maximize the impact of scarce resources and how efficiently managing congestion can improve decentralized system performance.

This chapter is organized as follows. Definitions and formal models are presented in Section 2.2. Section 2.3 highlights the contributions of this research and summarizes related literature. In Section 2.4, we introduce an efficient algorithm for finding user equilibrium solutions. Sections 2.5 and 2.6 analyze the worst- and best-case performance, respectively, of the user equilibrium solutions in the decentralized system. We discuss ways to improve the cost of equilibrium solutions in Section 2.7 and conclude in Section 2.8 with directions of current and future research.

2.2 Definitions and Models

The decentralized systems we study are related to congestion games and selfish routing problems that are explored in the disciplines of game theory, computer science, transportation engineering, and operations research. Here we introduce definitions from these fields and present formal mathematical models of the problem we study.

2.2.1 Definitions

The following definitions are used in classifying decentralized systems. *Unweighted* problems consider decentralized decision makers with identical demand, while those with *weighted* users admit differing demands. In the *nonatomic* case, each user controls an infinitesimal amount of flow which can be split over any number of paths in the network; in this case, only the cumulative effect of customers has measurable impact on the system. In contrast, in *atomic* problems the decisions of each individual have a non-negligible effect on overall system performance. In the atomic case, there are two scenarios: individuals' demand may be *splittable* or *unsplittable*.

Important concepts in the study of decentralized systems are those of Nash equilibria, price of anarchy, and price of stability. A Nash equilibrium is a solution in which no customer can improve his selfish objective by unilaterally changing his strategy; such solutions are said to be stable. However, equilibrium solutions can be more costly than centralized optimal solutions, which are chosen to minimize total system-wide cost. A measure called the price of anarchy is used to quantify the worst-case performance degradation that results from lack of centralized control. First defined in [56] and [74], it is the ratio of the total cost of the **worst** Nash equilibrium to the cost of a centralized optimal solution. The price of stability is a similar ratio under the **best** Nash equilibrium. In effect, the price of stability quantifies the minimum efficiency loss a centralized planner will incur by proposing a stable solution from which no decentralized decision maker will deviate. Throughout the chapter, when we refer to the price of anarchy (stability) for a class of problems, we mean the maximum price of anarchy (stability) among all problems in the class.

The decentralized systems we study fall into a class of problems called congestion games, first defined by Rosenthal [79], in which a set of self-interested players choose from among a set of resources. The cost of using each resource is a non-decreasing function of the number of players choosing the resource; it may be the same for all users or it may be player-specific. The cost to an individual player of a given solution is the sum of the costs of each of the resources she has chosen. Congestion games in which all players have identical choices available to them are called *symmetric* congestion games; those in which players' choices differ are *asymmetric*. Network congestion games are those in which the actions of the players can be represented as paths in a network (N, E), with starting and ending nodes a_i and b_i for each player i. Each player chooses a set of resources, represented by the edges in the network, such that the resources chosen by i comprise a path in the network from a_i to b_i. The delay, or latency, function on each edge is a non-decreasing function of the number of players using the edge. Selfish routing problems, which have been studied in particular for road networks and computer networks, are frequently modeled as network congestion games.

2.2.2 The Model

The problem we study is modeled on a graph, $G = (N, E)$. The node set, N, is composed of three subsets: a set of user locations $\mathcal{N}$ indexed by $i = A, B, \ldots, n$; a set of facility locations $\mathcal{M}$ indexed by $j = 1, 2, \ldots, m$; and a dummy sink node t. Edges in E are directed and belong to one of two subsets, E_f and E_t. Arcs $(i, j) \in E_f$ represent travel from user locations to facility locations. Arcs $(j, t) \in E_t$, also referred to as *sink arcs*, connect each facility location j to t and represent service at facility j. Costs on arcs in E represent costs experienced by users who traverse the arcs and are non-decreasing functions of the number of users. We consider constant latencies $d_{ij} \geq 0$ on the travel time arcs (i, j) and linear latency functions x_{jt} on the sink arcs (j, t), where x_{jt} is the total number of players using arc (j, t). The values used for these two cost components are converted into a common unit for use in the models.

The actions of the decentralized users are represented as paths in the network, with the path for player i beginning at $i \in \mathcal{N}$, visiting exactly one facility location $j \in \mathcal{M}$, and ending at the sink t. A solution consists of one action for each player and can be completely specified by binary variables x_{ij} for each i and j, where x_{ij} is 1 if player i is served at facility j and 0 otherwise. Using this notation, the latency x_{jt} of each sink arc (j, t) is given by $x_{jt} = \sum_{i=1}^{n} x_{ij}$. For simplicity, in the remainder of the chapter we drop the t from the subscript because all sink arcs terminate at the common sink.

The flow in the network is unweighted, unsplittable, and atomic. Flow is *unweighted* because each customer has identical demand, that is, must be served by exactly one facility. The *unsplittable* characteristic indicates that each customer must choose a single path. *Atomic* flow means that each customer's decision has a non-negligible effect on the overall system performance.

Using this modeling framework, we examine the problem of assigning customers to facilities both from the perspectives of individual customers seeking to minimize their own costs and of a centralized planner aiming to minimize the system-wide cost.

2.2.2.1 The Individual's Problem

From the individual's perspective, we consider a class of objective functions capturing travel time, facility congestion, and weights related to congestion. We introduce the Facility-Specific Congestion Weights Problem (FSCWP), which focuses on congestion weights that are associated with facilities. This is modeled using a scalar, $\alpha_j \geq 0$, applied to the true congestion value at each facility j. The scalar is identical for all individuals in the system. Each individual seeks to minimize his travel time plus weighted congestion. An α_j value of 1 indicates that the decentralized users value congestion in their own objective functions at the same level that the centralized planner does in the system-wide objective. An α_j value greater than 1 means that decentralized decision makers place added weight on congestion experienced at facility j in comparison to the centralized planner, while a value less than 1 means that less weight is placed on congestion. If $\alpha_j = 0$, the congestion impact at facility j is not included in individuals' objective functions; facility j is evaluated based solely on the travel time individuals experience in visiting it.

Given this framework, an equilibrium solution is one in which the following condition holds for all individuals:

Definition 1 *Equilibrium Condition for Customer i:*

$$d_{ij} + \alpha_j \sum_{p=1}^{n} x_{pj} \leq d_{ik} + \alpha_k \left(\sum_{p=1}^{n} x_{pk} + 1 \right) \quad \forall\ k \neq j. \tag{1}$$

Here, j is customer i's facility in the solution being considered, while $x_{pj} = 1$ if customer p is served at facility j and 0 otherwise. The value α_j is the congestion weight associated with facility j.

In other words, when the full status of the system and the choices of each individual are revealed, no individual has incentive to deviate from an equilibrium solution because he cannot improve his objective by unilaterally changing his choice of facility. This context gives rise to an asymmetric network congestion game. Finding equilibria in general networks of this type is *PLS*-complete [33]. However, the problem is also in the class of congestion games with player-specific latency functions studied by Milchtaich [67], who showed that an equilibrium can be found efficiently using a specialized algorithm to prevent cycling infinitely through non-equilibrium solutions. As we will see, the structure of the FSCWP precludes this cycling behavior and admits a more straightforward approach for finding an equilibrium than that described in [67].

2.2.2.2 The Planner's Problem

In contrast to the preceding individuals' problems in which many decision makers seek to optimize selfish objectives, the planner's problem assumes that a single centralized entity can allocate customers to facilities to minimize the total cost experienced by the entire system. In our problem, the prices of anarchy and stability are measures of the additional congestion and/or travel time incurred when customers choose based on a selfish objective in comparison to the minimum cost that could be achieved by assigning customers to facilities using a central authority. To determine the prices of anarchy and stability, we develop a model for a centralized planner that serves as a benchmark for the decentralized systems.

The centralized planner's problem is given by:

$$\text{Minimize} \quad \sum_{i=1}^{n}\sum_{j=1}^{m} d_{ij}x_{ij} + \sum_{j=1}^{m}\left(\sum_{i=1}^{n} x_{ij}\right)^2 \tag{2}$$

$$\sum_{j=1}^{m} x_{ij} = 1 \qquad \forall i \tag{3}$$

$$x_{ij} \in \mathbb{B} \qquad \forall i, j \tag{4}$$

Despite the fact that individuals may make decisions based on facility-specific weights on congestion, the centralized planner seeks to minimize the sum of actual travel and waiting times incurred by all users, as shown in expression (2). Such an approach, which employs a linear cost for each individual, is common in the framework of congestion games. Moreover, while it is reasonable that individuals consider congestion in some form, it is unlikely that they utilize queuing functions or other more complicated expressions. The centralized objective function is thus chosen to maintain similarity between the planner's objective and the individual objectives. In the formulation of the centralized planner's problem, constraint (3) simply requires that every individual be served at exactly one facility. The centralized problem is modeled as a convex cost flow problem, and thus is polynomially solvable.

2.3 Contributions and Related Literature

In what follows, we describe the contributions of this research and place it in the context of related literature.

2.3.1 Contributions

This chapter presents fundamental contributions to the theory of managing systems with decentralized decision makers. We introduce the Facility-Specific Congestion Weights Problem (FSCWP), in which individuals choose to visit one facility among a set of facilities opened by a centralized planner. We consider individual objectives that include travel time, congestion, and weights on congestion. Travel time and congestion are natural decision objectives on the part of individuals, but the inclusion of facility-specific weights on congestion is novel and enables the representation of a broader range of scenarios that are important in practice. We present new results on the complexity of finding equilibrium solutions, bounds on the quality of these solutions, and methods for improving decentralized performance in these systems.

The decentralized objective structure of the FSCWP gives rise to an asymmetric network congestion game. Finding Nash equilibria in general instances of such games is PLS-complete [33] and greedy algorithms can cycle infinitely even in cases where equilibria can be found efficiently [67]. However, we prove that our problem admits a polynomial time algorithm for finding an equilibrium solution, regardless of the values of congestion weights. The proof of this result also implies that greedy approaches cannot succumb to infinite cycles when applied to the FSCWP.

We provide new bounds on the price of anarchy for the FSCWP. The results are classified by the congestion weight values allowed in the objective function. When congestion weights are allowed to take on any values greater than 0, we show that the price of anarchy can be arbitrarily high. In contrast, constant bounds have been found for both atomic [8] and nonatomic [82] cases when congestion weights are not a component of the objective. When congestion weights are restricted to be greater than or equal to the true facility congestion, we provide a tight lower bound on the price of anarchy as well as an upper bound, each of which is a function of the greatest weight. When the congestion weight associated with all facilities is equal to the true congestion, we show that the price of anarchy is at least 2. This problem is a special case of that considered in [8], which means that 2.5 is an upper bound on the price of anarchy. Finally, when decentralized decision makers do not consider congestion but instead make decisions based only on travel time, we prove that the price of anarchy is $O(m)$, where m is the number of facilities.

In addition to the price of anarchy, we also consider the price of stability for the FSCWP. When individual objectives consider only travel time, the price of stability is the same as the price of anarchy, $O(m)$. We prove that the price of stability for a given network is no greater when individuals consider travel time plus true congestion than when they consider only travel time. We also illustrate that, for general values of congestion weights, the price of stability can be worse than when congestions weights are either all 0 (travel time only) or all 1 (travel time plus true congestion). These results show that even the best equilibrium solutions can be costly, and they point to the need for designing decentralized systems in ways that lead to better outcomes.

Having demonstrated that the decentralized systems in question can perform poorly, we investigate ways to improve them. We first demonstrate that it is always possible to find a facility-specific congestion weight vector for which the centralized optimal solution is also an equilibrium solution. We refer to these weights as *equilibrium-obtaining* and go on to characterize the set of such congestion weight vectors by examining what their values *can* and *must* be. Finding a single equilibrium-obtaining congestion weight vector is important, because it indicates that by changing the importance that individual decision makers place on congestion we can improve the decentralized solution. However, we also investigate equilibrium-obtaining congestion weight vectors that optimize particular policy objectives. For example, planners may wish to improve the decentralized system in a way that distributes resources equitably or that minimizes the investment needed for additional resources. To accomplish these goals, we develop optimization models to find the equilibrium-obtaining congestion weight vector that minimizes the range of weights across all facilities or that minimizes the change from an initial congestion weight vector, respectively. We show that solutions to these problems can be found efficiently using an algorithm for the generalized circulation problem presented in [102]. Equilibrium-obtaining congestion weights thus represent a mechanism for coordinating this class of decentralized logistics systems. Since the assignment of demands to service providers is a component of many several classes of logistics problems, such as facility location, the mechanism we introduce can also inform the study of decentralized versions of these problems.

2.3.2 Related Literature

Previous work on decentralized systems has examined the existence of Nash equilibrium solutions, the computational complexity of finding them, and their quality in comparison to centralized optimal solutions. In summarizing key results from the literature on congestion games, we first turn our attention to the case of atomic, unsplittable, and unweighted flow. This is the setting of the FSCWP. Rosenthal [79] shows that every congestion game of this type, whether or not actions are represented as paths in a network, has a pure Nash equilibrium (that is, each player chooses exactly one strategy). This is in contrast to a mixed equilibrium, in which each player employs a probability distribution over his set of feasible strategies. Researchers examine the complexity of finding Nash equilibrium solutions from two angles: first, determining whether *any* algorithm exists by which equilibria can be found efficiently and second, determining whether a *best-reply path* can reach an equilibrium efficiently from an arbitrary starting point. In the latter approach, the sequence of solutions is called a best-reply path because at each step, a single deviator switches to a strategy that minimizes his cost with respect to the other players' strategies – he switches to his best reply. This can be characterized as a greedy, or myopic, algorithm. In the context of congestion games with atomic, unsplittable, and unweighted flow, Fabrikant et al. [33], show that finding Nash equilibria is PLS-complete except in the symmetric case. Even when it is possible to find a Nash equilibrium efficiently, best-reply paths can be cyclic [67], meaning that their length is unbounded and this approach will not produce an equilibrium solution efficiently. [1] and [52] identify conditions under which best-reply paths are polynomially bounded in length. Milchtaich [67] shows that for player-specific linear latency functions, although infinite best-reply paths may exist, there is always a path connecting an arbitrary initial point to an equilibrium; the length of this path is polynomially bounded in the size of the strategy space. The quality of equilibrium solutions is also an important question. Awerbuch et al. [8], show that for linear latency functions that are identical for all users, unweighted and unsplittable demand, and pure strategies, the price of anarchy in a network congestion game is 2.5. Christodoulou and Koutsoupias [18] prove analogous results independently and also provide bounds for more general latency functions. In a separate paper [17], the same authors provide lower and upper bounds on the price of stability for such networks, demonstrating that the best equilibrium solution is between $1 + \sqrt{3}/3$ and 1.6.

Others [35, 36, 61, 25, 2] have studied congestion games in which individuals have weighted, rather than identical, demand. In this case, a pure Nash equilibrium is no longer guaranteed to exist. Researchers in this area establish conditions that ensure the existence of pure equilibrium solutions, or in the absence of pure equilibria, study mixed equilibrium solutions. The reader is referred to the survey by Kontogiannis and Spirakis [55] and the references therein for additional results on the existence of pure Nash equilibria, the efficiency of constructing them, and prices of anarchy and stability of atomic selfish routing in networks with both weighted and unweighted users.

Much work has been done on the nonatomic case; see, for example, the survey by Roughgarden [81] and the references therein. In the seminal paper by Roughgarden and Tardos [82], they prove that the price of anarchy for nonatomic flow and linear latencies is 4/3, while for general latency functions the value is unbounded. Correa et al. [21], provide simplified proofs of the Roughgarden and Tardos results for networks with separable cost functions. Additional work on nonatomic congestion games has examined other types of cost functions [21, 22], uncertainty in cost parameters [72], and fairness in individual user costs incurred in an equilibrium solution [22]. When flow is atomic but splittable, Roughgarden [80] shows that the bounds on the price of anarchy in the nonatomic case still hold when demand is unweighted. However, when players have weighted demand, Cominetti et al. [20] prove upper and lower bounds on the price of anarchy that are strictly higher than those in the nonatomic case.

In addition to developing bounds on decentralized system performance and determining the complexity of finding equilibrium solutions, researchers have investigated how to design systems to reduce the negative impact of decentralized decision making. Roughgarden's survey [81] highlights three of these in the nonatomic case: increasing network capacity, routing part of the traffic centrally, and imposing taxes on links. Anshelevich et al. [5], study the design of a network in which users share the link construction costs. They are among the first to define the price of stability, and they provide a bound on its value for this network design game. Roughgarden and Tardos [83] seek games for which the decentralized solution is approximately optimal, while Chen et al. [16], design cost-sharing protocols to minimize the prices of anarchy and stability. Paz and Peeta [75] provide information to drivers in a traffic network to influence behavior toward route choices that more closely approximate the system-optimal solution. Although their model is not within the congestion game framework, their results describe an approach for improving decentralized systems by accounting for individual behavior.

We conclude the literature summary by briefly examining related work in the area of transportation and logistics network management, which is the motivating application domain for our research. The assignment of demands to facilities or service providers is a component of many logistics problems, especially in the area of facility location. The reader is referred to [24] and [27], as well as the work cited therein, for thorough reviews of discrete network location models and location problems in transportation, respectively. In these models, either a centralized planner determines the assignment of demands to facilities or, if individuals choose, distance is their principal criterion. A notable exception is the work of [7], in which a firm's facility location decisions are made under the assumption that customers will choose a facility based on service price and transportation cost, both of which are functions of the location and the number of other customers choosing the facility. Revelle et al. [78] provide a recent review of facility location models in which they cite the need for additional work that incorporates congestion and explicitly models customers' demands within the system.

Decision making by decentralized, collaborative agents has been modeled in several transportation and logistics application areas. The objective of much of this research is the development of mechanisms that facilitate cooperation toward an end that is beneficial for all parties but that still optimizes the individual agents' objectives. Examples include alliances in passenger air [98, 105], air cargo [48], sea cargo [4, 14], and truckload transportation [73] industries; the management of air traffic flow [100, 63]; cooperative facility location [39]; retail transshipment coordination [42]; and general multi-commodity flow frameworks [3, 41].

2.4 Complexity of Finding an Equilibrium

In this section, we explore the complexity of identifying equilibrium solutions to FSCWP. Recall that such equilibria exist [79], but finding one can be challenging in general network congestion games [33, 67]. The structure of the network for the FSCWP, however, admits a straightforward polynomial time algorithm for finding a Nash equilibrium solution. Before proceeding with the proof for the general case, we formalize an observation for the case in which individuals consider only travel time in their objectives.

Remark 1 *When $\alpha_j = 0$ for all j, that is, when decentralized decision makers consider only travel time, finding an equilibrium solution is trivial. Each individual simply visits the closest facility.*

We now present the result for more general α vectors.

Theorem 1 *A pure strategy Nash equilibrium for the FSCWP can be found in polynomial time.*

Proof. The proof consists of three parts. We first present a transformation of the original network to a minimum cost flow problem on a related network. Then we show that all combinations of customer choices can be represented as feasible flows in the transformed network. Finally, we prove that the minimum cost flow on this network represents a solution that is the global minimizer of a particular potential function, and thus a Nash equilibrium.

We first present the network transformation, which is illustrated in Figure 1 for a small example network. Recall the network depiction of the original problem, with arcs (i, j) from each customer i to each facility j and corresponding costs d_{ij}. Arcs (j, t) from each facility j to the sink t have cost x_j, where x_j is the total number of customers served at facility j. Every customer node i has net supply of 1 unit, every

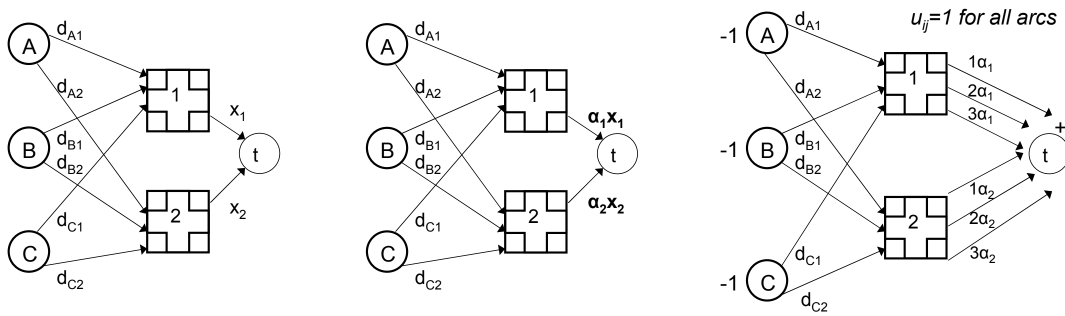


Figure 1: Original, Perceived, and Minimum Cost Flow Networks.

facility node j has 0 supply, and the sink node t has net demand of n units, where n is the total number of customers.. The central optimal solution to the FSCWP problem corresponds to a minimum cost flow in this network. In Figure 1, the original network is pictured on the left. The network for the decentralized decision makers is identical, with the exception that the perceived cost on arc (j, t) from each facility j to the sink t is $\alpha_j x_j$. The perceived network is shown in the center of Figure 1. Given this structure, we create a related network as follows. First, replace each sink arc (j, t) with n copies of the arc, each with capacity 1. The costs $c_{j_k,t}$ on duplicate arcs $(j_1, t), (j_2, t), \ldots, (j_n, t)$ are $c_{j_k,t} = k \cdot \alpha_{jk}$ for all $j = 1, \ldots, m$ and $k = 1, \ldots, n$. All other arcs remain the same and have the same costs as the original network, and the supplies at each node are identical to those in the original network. The result of this transformation is depicted on the right of Figure 1. The transformation takes $O(mn)$ effort.

We now prove that an assignment of customers to facilities is feasible if and only if it corresponds to a feasible flow in the transformed network. Suppose we have a feasible assignment of customers to facilities; that is, each customer is served at exactly one facility. By the construction of the transformed network, we can represent this assignment as the composite of n unit flows, one from each customer to the facility and then to the sink. This composite flow clearly satisfies flow balance constraints, and since there are n arcs from each facility to the sink we satisfy flow bound constraints as well. Therefore, the assignment can be represented as a feasible flow in the transformed network. Now suppose that f is a feasible integer flow in the transformed network. This implies that 1 unit is supplied from each customer and that n units terminate at the sink without violating flow bounds. We need only look at the flow on arcs (i, j) and assign customer i to the facility j for which the flow from i to j is 1. Thus all customers are assigned to exactly one facility, and the feasible flow corresponds to a feasible assignment of customers to facilities.

Finally, we employ a potential function argument similar to that presented in [79] to show that the minimum cost flow in the transformed network represents a Nash equilibrium for the original problem. For a particular solution s, let x_{ij}^s be the binary variable indicating whether i is served at j, x_j^s is the total number of individuals served at facility j, and α_j is the congestion weight associated with facility j. The potential function that establishes the result is

$$\phi(s) = \sum_{i=1}^{n} \sum_{j=1}^{m} d_{ij} x_{ij}^s + \sum_{j=1}^{m} \sum_{y=0}^{x_j^s} \alpha_j y.$$

(Note that y in this expression just serves as the index of summation, so that the right-most term in the potential function is $\alpha_j \cdot (0 + 1 + \ldots + x_j^s)$ for each j.) Clearly, the cost of a feasible flow s in the transformed network is given by $\phi(s)$. Suppose the change from solution s to s' involves a move by a single individual i from facility j to facility k and that the move improves i's objective function. The change in i's objective is equal to the change in the potential function value:

$$\begin{aligned}\phi(s) - \phi(s') &= \left(\sum_{i=1}^{n} \sum_{j=1}^{m} d_{ij} x_{ij}^s + \sum_{j=1}^{m} \sum_{y=0}^{x_j^s} \alpha_j y \right) - \left(\sum_{i=1}^{n} \sum_{j=1}^{m} d_{ij} x_{ij}^{s'} + \sum_{j=1}^{m} \sum_{y=0}^{x_j^{s'}} \alpha_j y \right) \\ &= (d_{ij} + \alpha_j x_j^s) - (d_{ik} + \alpha_k x_k^{s'}) = cost_i(s) - cost_i(s').\end{aligned}$$

Since the move from s to s' improves i's objective function, $cost_i(s) > cost_i(s')$. The potential function decreases by the same amount as does player i's objective function as a result of the move. Thus, every improving move on the part of an individual results in an equal decrease in the potential function value. Rosenthal [79] proved that any potential function local minima corresponds to a Nash equilibrium. By solving the minimum cost flow problem on the transformed network, we obtain a global minima of the function, which is well-defined since the potential function is non-negative. Because the global minima is certainly a local minima, and because we have shown that a feasible flow corresponds to a feasible assignment of customers to facilities, the solution to the minimum cost flow problem is a Nash equilibrium of the original problem.

The network transformation is polynomial and the minimum cost flow problem can be solved in polynomial time, implying that a Nash equilibrium for the FSCWP can be found in polynomial time. □

This algorithm extends the results of [33] by showing that an equilibrium for this particular asymmetric congestion game can also be found in polynomial time using minimum cost flow techniques. Previously, this had only been proven for symmetric networks. This is a more straightforward algorithm than that presented in [67] for general player-specific linear latency functions, which is possible due to additional latency function structure in this problem. The existence of a potential function also implies the next result.

Corollary 1 *Infinite best-reply paths cannot occur in the FSCWP.*

Proof. Monderer and Shapley [68] proved that the existence of a potential function implies the finite improvement property, meaning that infinite best-reply paths cannot occur. □

In Chapter 3, we describe our analysis of a recent emergency response scenario, in which we implement this algorithm using data from a large-scale network with individual decision makers.

2.5 Price of Anarchy

In this section, we present bounds on the price of anarchy under several different congestion weight scenarios. Recall that the price of anarchy is the ratio of the costs of the worst equilibrium solution and the central optimal solution. Quantifying this value is an important task in designing and managing decentralized systems. Ultimately, system planners seek ways to minimize the impact of decentralized decision making by reducing the price of anarchy. Here, we consider four cases distinguished by their congestion weight values. The first and most general case is that in which α_j can take on any value greater than 0 for a given facility j. In the second case, we restrict α_j to be at least 1 for every facility, that is, congestion is at least as important to individuals as travel time. Thirdly, we present results for systems in which $\alpha_j = 1$ for all j. This scenario is one in which decentralized decision makers place the same weight on congestion at the facilities that the system planner does; in essence, they perceive the true congestion. Finally, we examine the case in which $\alpha_j = 0$ for all j. Here, individuals choose facilities on the basis of travel time alone and do not consider congestion at any facility. The reader is referred to Appendix B.2 for the description of an approach we developed to study the price of anarchy computationally.

2.5.1 Case 1: Congestion Weights Greater than 0

We begin with the case in which $\alpha_j > 0$ for all j. In this broad scenario, the decentralized system can perform quite poorly in comparison to a centralized system.

Theorem 2 *The price of anarchy in systems with $\alpha_j > 0$ for all j is at least $\left\lfloor \frac{\alpha_{max}}{\alpha_{min}} \right\rfloor$, where α_{max} and α_{min} denote the maximum and minimum α_j values in the system, respectively.*

Proof. Consider a network consisting of $n = \left\lfloor \frac{\alpha_{max}}{\alpha_{min}} \right\rfloor$ customers and $m = n + 1$ facilities. Distances between customer i and facilities $1, \ldots, n$ are $d_{i,1} = d_{i,2} = \ldots =$

$d_{i,n} = 0$ for all i, while that to facility $n+1$ is $d_{i,n+1} = \alpha_{max} - n\alpha_{min}$ for all customers i. Similarly, the facility congestion weights are $\alpha_1 = \alpha_2 = \ldots = \alpha_n = \alpha_{max}$ while $\alpha_{n+1} = \alpha_{min}$. The highest-cost equilibrium solution has all customers at facility $n+1$, with the perceived cost to each customer given by the sum of his distance, $\alpha_{max} - n\alpha_{min}$, plus his weighted congestion, $n\alpha_{min}$, which is equal to α_{max}. This is an equilibrium solution because the perceived cost of switching to another facility is also α_{max}. However, the true cost of this solution is $n(\alpha_{max} - n\alpha_{min} + n)$. The optimal solution serves one customer at each of the first n facilities, incurring a total cost of n. Therefore, we have

$$\text{Price of Anarchy}(\alpha > 0) = \frac{n(\alpha_{max} - n\alpha_{min} + n)}{n} = \alpha_{max} - n\alpha_{min} + n \geq n = \left\lfloor \frac{\alpha_{max}}{\alpha_{min}} \right\rfloor,$$

where the inequality follows from the fact that $\alpha_{max} - n\alpha_{min} \geq 0$ by the choice of n. $\square$

As a consequence of Theorem 2, it is possible to construct decentralized systems with arbitrarily large prices of anarchy by making the ratio between α_{max} and α_{min} suitably large. The intuition behind this observation is that large discrepancies in the weights associated with congestion at different facilities can lead to decentralized choices that drastically increase system-wide costs. However, by examining more restricted ranges for α values, we hope to obtain better bounds on the price of anarchy.

2.5.2 Case 2: Congestion Weights Greater than or Equal to 1

We next examine the case in which $\alpha_j \geq 1$ for all j. In these systems, decentralized decision makers perceive at least the true congestion value, although they may assign greater weight congestion. In this scenario, we are able to provide both upper and lower bounds on the price of anarchy.

Theorem 3 *The price of anarchy in systems with $\alpha_j \geq 1$ for all j satisfies*

$$\alpha_{max} + 1 \leq \textit{Price of Anarchy} \leq 2.5\alpha_{max},$$

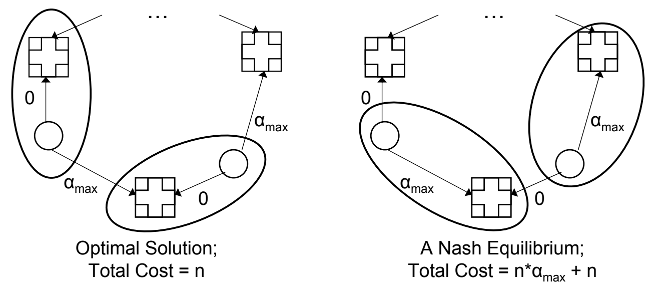


Figure 2: Lower Bound on Price of Anarchy when $\alpha_j \geq 1$ for all j.

where α_{max} denotes the maximum α value in the system.

Proof Sketch. The full proof is provided in Appendix A and consists of two parts. We first demonstrate the lower bound by providing a network structure, illustrated in Figure 2, for which the lower bound is tight for any value of $\alpha_{max} \geq 1$ and for any number of customers, n, and facilities, m. We then prove the upper bound by establishing relationships between equilibrium costs to individuals, system-wide cost of equilibrium solutions, and central optimal solution cost.

2.5.3 Case 3: Congestion Weights Equal to 1

Systems in which $\alpha_j = 1$ for all j constitute of a subset of those considered in Case 2. In this setting, individuals place the same importance on congestion at each facility as does the centralized planner. The results from Section 2.5.2 have immediate implications for this scenario.

Corollary 2 *If $\alpha_j = 1$ for all facilities j, then the price of anarchy satisfies*

$$2 \leq \textit{Price of Anarchy} \leq 2.5.$$

Proof. The result follows from substituting $\alpha_{max} = 1$ into the result from Theorem 3. □

There are a number of network structures for which we have proven that the price of anarchy is exactly 2 when $\alpha_j = 1$ for all j; these are described in Appendix

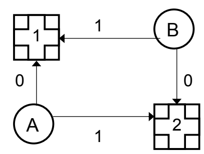


Figure 3: Price of Anarchy Can Be Worse When $\alpha_j = 1$ than When $\alpha_j = 0$ for all j.

B.1.1. We further conjecture that this is the worst-case bound on the performance of decentralized systems for general network structures when all the α's are 1. The proof of this conjecture remains a component of our ongoing research.

It may seem that decentralized performance should be better in systems in which individual and system-wide objectives perceive congestion in the same way than in systems where individuals assign other weights to congestion. However, we find that this is not always the case.

Remark 2 *There are networks for which the price of anarchy is greater when individuals choose based on travel time and true congestion than when they choose based solely on travel time.*

Proof. Consider the family of networks represented by Figure 3 with $\alpha_{max} = 1$ and the same networks with $\alpha_{max} = 0$.

When $\alpha_{max} = 1$, (that is, individuals choose based on travel time and true congestion), the price of anarchy is 2, because it is an equilibrium when both individuals choose their more distant facility. However, when $\alpha_{max} = 0$, the only equilibrium solution is the optimal solution, where both individuals visit their closest facility, and the price of anarchy is 1. This structure can be replicated for any number of customers n and facilities m, where $n = m$. $\square$

This result demonstrates that even when decentralized decision makers incorporate both travel time and true congestion in their objectives, this does not always result in lower system-wide costs than a simpler objective that considers only travel time.

2.5.4 Case 4: Congestion Weights Equal to 0

In this section, we quantify the performance of decentralized systems in which individuals seek to minimize their own travel time. The congestion weight α_j associated with each facility j is 0. Here, individuals either do not know the system status and choices of other customers or they do not incorporate this information into their decision making. Before presenting price of anarchy results for this scenario, we formalize some observations that will be useful in the subsequent discussion.

Remark 3 *Let D_d be the total travel time incurred by all customers in a decentralized solution in which customers choose based only on travel time and D_c be the total travel time in a centralized optimal solution. Then $D_c \geq D_d$.*

Proof. Suppose, for contradiction, that the total travel time in the centralized optimal solution is less than that in a decentralized solution. Then there must be at least one customer who experiences a shorter travel time. However, since customers choose on nominal travel time, this customer would have chosen the facility with shorter travel time in the decentralized solution. □

Remark 4 *Let C_d be the total system congestion in a decentralized solution in which customers choose on nominal travel time and C_c be the total system congestion in a centralized optimal solution. Then $C_c \leq C_d$.*

Proof. Suppose, for contradiction, that $C_c > C_d$. The total cost of the decentralized solution is $D_d + C_d$ and that of the centralized optimal solution is $D_c + C_c$. By Lemma 3, $D_c \geq D_d$. But then $D_c + C_c > D_d + C_d$, contradicting the optimality of the centralized solution. □

Given the preceding observations, we are equipped to bound the price of anarchy in the case in which individuals choose solely on travel time.

Theorem 4 *The price of anarchy for general networks in which $\alpha_j = 0$ for all j is $O(m)$.*

Proof. We begin by providing an upper bound for the cost of the equilibrium solution and a lower bound for that of the centralized optimal solution. We know that in the case in which all α_j's are 0, the total travel time in an equilibrium solution is equal to the sum over all the customers of the travel time from each customer to his closest facility. We again represent this value as D_d. The worst congestion in any possible scenario is n^2, which occurs when all customers choose the same facility. We thus obtain the following upper bound on the cost of a decentralized solution:

$$\text{Total Cost of Equilibrium Solution} \leq D_d + n^2.$$

This upper bound is tight for networks in which all individuals choose the same facility in an equilibrium solution.

To obtain a lower bound on the centralized cost, observe that the minimum possible congestion in any network occurs when individuals are split equally among facilities, resulting in a total congestion of $\frac{1}{m}n^2$. Letting D_c represent the total travel time in the centralized solution and invoking Lemma 3, we have a lower bound on the cost of the centralized solution:

$$\text{Total Cost of Centralized Solution} \geq D_c + \frac{1}{m}n^2 \geq D_d + \frac{1}{m}n^2.$$

Note that this bound is tight in networks for which the centralized optimal solution is to split customers equally between facilities but for which $D_c = D_d$. That is, the centralized solution achieves the minimum congestion without incurring additional travel time beyond that experienced in the equilibrium solution in which individuals choose their closest facility.

Combining these two bounds, we have:

$$\text{Price of Anarchy}(\alpha = \mathbf{0}) = \frac{\text{Total Cost of Worst Equilibrium}}{\text{Total Cost of Centralized Solution}} \leq \frac{D_d + n^2}{D_d + \frac{1}{m}n^2}.$$

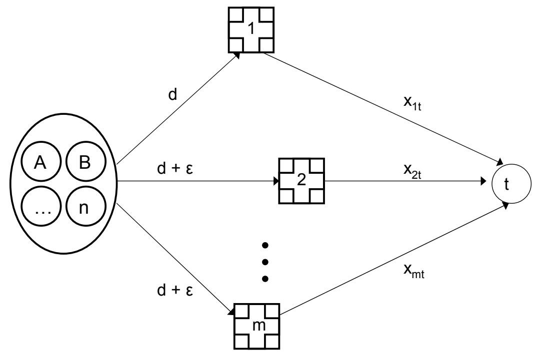


Figure 4: Price of Anarchy when $\alpha = \mathbf{0}$.

Therefore, the Price of Anarchy($\alpha = \mathbf{0}$) $\leq$ $O(m)$.

To see that the result is tight, consider the network in Figure 4. Each customer's travel time to Facility 1 is d units, while the time to reach Facilities $2, \ldots, m$ is $d + \epsilon$ units. An equilibrium solution is one in which all customers visit Facility 1. As ϵ goes to 0, the central optimal solution is to split the customers equally among all facilities. For any values of m and n, we obtain

$$\text{Price of Anarchy}(\alpha = \mathbf{0}) = \frac{nd + n^2}{nd + n\epsilon + \frac{1}{m}n^2} = O(m). \quad \square$$

Another way to view this result is that when $\epsilon = 0$, the network in Figure 4 simultaneously satisfies the cases for which the upper and lower bounds presented in the proof are tight.

We document additional results concerning special cases of the FSCWP when $\alpha_j = 0$ for all j in Appendix B.1.2.

In this section, we have quantified the performance of the worst equilibrium solutions under several different congestion weight scenarios. Although this performance can be bad, we also seek to understand whether there are alternative equilibrium solutions that perform better.

2.6 *Price of Stability*

Recall that the price of stability is the ratio of the costs of the best equilibrium solution and the central optimal solution. Here we examine the price of stability under different congestion weight scenarios, in particular comparing this value across the scenarios.

2.6.1 Price of Stability: Congestion Weights Equal to 0

We begin with the scenario in which $\alpha_j = 0$ for all j because results for other scenarios are built upon it. In Section 2.5.4 we presented the price of anarchy results for this case. The network in Figure 4, for which the price of anarchy result is tight, immediately implies the following result for the price of stability.

Theorem 5 *The price of stability in the case in which $\alpha_j = 0$ for all j is at most $O(m)$.*

Proof. Recall that the price of stability is the ratio of the cost of the best equilibrium solution to that of a centrally optimal solution. In the network in Figure 4, for any $\epsilon > 0$, the single equilibrium solution is the one in which all customers choose Facility 1. We have seen that the ratio of the cost of this equilibrium to that of the centrally optimal solution is $O(m)$. The ratio can clearly be no greater, because from the proof of Theorem 4 we have that $O(m)$ is an upper bound on the ratio of the costs of the equilibrium and centrally optimal solutions. □

2.6.2 Price of Stability: Congestion Weights Equal to 1

In this section, we examine the scenario in which $\alpha_j = 1$ for all j. Recall that [17] provide nearly equal lower and upper bounds on the price of stability for this case, namely that the value is between $1+\sqrt{3}/3$ (about 1.577) and 1.6. In contrast to our results for the case in which $\alpha_j = 0$ for all j, the best equilibrium solutions can thus be quite good. This illustrates that the weight associated with facility congestion can significantly influence decentralized system outcomes. As Corollary 2 establishes, there are networks in which the $\alpha = 1$ congestion weight framework can result in more costly solutions than the framework in which α_j's are 0 and individuals consider only travel time. Despite this fact, we prove that the best equilibrium under the $\alpha = 1$ framework has cost no greater than the best equilibrium under the $\alpha = 0$ framework. Before presenting the proof, we state two preliminary lemmas. The proofs of the lemmas are included in Appendix A.

Lemma 1 *Let x be a solution in which customer i is served at facility j and let x_j be the total number of customers at j excluding i. Similarly, let x_k be the total number of customers at k. Then the net effect on the total system cost of moving customer i from facility j to facility k is $2x_k - 2x_j + d_{ik} - d_{ij}$.*

Lemma 2 *Given any customer i and any solution that satisfies i's Equilibrium Condition (Definition 1) under $\alpha = 0$ but not under $\alpha = 1$, then the single change in which customer i is moved to a facility that satisfies the latter condition strictly decreases the system-wide cost.*

We are now equipped to present the theorem that relates the costs of the best equilibria under the scenarios in which congestion weights are 0 and 1, respectively.

Theorem 6 *For $\alpha = 0$, let $x^*(\alpha = 0)$ be the solution with lowest total cost in which the Equilibrium Condition is satisfied for all customers; for $\alpha = 1$, let $x^*(\alpha = 1)$ be the solution with lowest total cost in which the Equilibrium Condition is satisfied for all customers; and let $z^*(\alpha = 0)$ and $z^*(\alpha = 1)$ be their corresponding system cost values. Then $z^*(\alpha = 1) \leq z^*(\alpha = 0)$.*

Proof Sketch. The full proof is presented in Appendix A. For clarity of exposition, we present only the main ideas here. The proof consists of two parts. First, we begin at an arbitrary equilibrium solution under the $\alpha = 0$ framework and move customers one by one (possibly moving a customer more than once) such that the current move satisfies that customer's Equilibrium Condition under the $\alpha = 1$ framework. We show that the cumulative change in system cost that results from these moves is strictly negative using the results from Lemmas 1 and 2. Second, we show that this iterative process reaches an equilibrium solution under the $\alpha = 1$ framework in a pseudo-polynomial number of steps under the assumption that travel times and α values are rational numbers. $\square$

Observe that the constructive proof of this result also provides an algorithm for constructing an equilibrium solution under the $\alpha = 1$ framework given any equilibrium under the $\alpha = 0$ framework, the latter of which is easy to find as per Remark 1. The result also implies the following corollaries.

Corollary 3 *The price of stability in any network in the scenario in which $\alpha_j = 1$ for all j is no greater than that in the same network in the scenario in which $\alpha_j = 0$ for all j .*

Proof. The price of stability is defined as follows:

$$\text{Price of Stability}(\alpha = 0) = \frac{z^*(\alpha = 0)}{z^*}$$

$$\text{Price of Stability}(\alpha = 1) = \frac{z^*(\alpha = 1)}{z^*}$$

Here, z^* is the total system cost of the central optimal solution. From Theorem 6 we have $z^*(\alpha = 1) \leq z^*(\alpha = 0)$, so

$$\frac{z^*(\alpha = 1)}{z^*} \leq \frac{z^*(\alpha = 0)}{z^*}.\square$$

Corollary 4 *Let $D^*(\alpha = 0)$ and $C^*(\alpha = 0)$ be the total travel time and total congestion in the best equilibrium solution when $\alpha_j = 0$ for all j, respectively. Let $D^*(\alpha = 1)$ and $C^*(\alpha = 1)$ be the total travel time and total congestion in the best equilibrium solution when $\alpha_j = 1$ for all j, respectively. Then $D^*(\alpha = 1) \geq D^*(\alpha = 0)$ and $C^*(\alpha = 1) \leq C^*(\alpha = 0)$.*

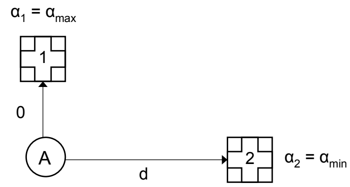


Figure 5: Price of Stability for $\alpha \geq \mathbf{0}$.

Proof. Clearly $D^*(\alpha = 1) \geq D^*(\alpha = 0)$ holds since $D^*(\alpha = 0)$ is achieved when each customer chooses a facility with minimum travel time. Then $C^*(\alpha = 1) \leq C^*(\alpha = 0)$ follows from the travel time relationship and the result of Theorem 6. □

We have thus characterized relationships between equilibrium solutions found by decentralized users under different selfish objectives. We have also seen that although the price of stability is no greater in the scenario in which $\alpha_j = 1$ for all j than when $\alpha_j = 0$ for all j, the former can still lead to poor performance.

2.6.3 Price of Stability: Congestion Weights Greater than or Equal to 0

Unfortunately, simple networks confirm that the price of stability under general congestion weight values can be worse than both of the more restricted frameworks, that is, when $\alpha_j = 0$ or when $\alpha_j = 1$ for all j.

Lemma 3 *The price of stability for a given network in systems with congestion weights $\alpha_j \geq 0$ for all j can be worse than that in the same network when $\alpha_j = 0$ for all j or when $\alpha_j = 1$ for all j.*

Proof. Consider the simple network in Figure 5. The distance from Customer A to Facility 1 is 0, while that to Facility 2 is $d > 0$. The central optimal solution sends the single customer to Facility 1, incurring a total cost of 1. This is also the only equilibrium solution for the case with $\alpha_1 = \alpha_2 = 0$ and for the case with $\alpha_1 = \alpha_2 = 1$. However, for any values of $\alpha_1 = \alpha_{max}$ and $\alpha_2 = \alpha_{min}$ such that $\alpha_{min} + d < \alpha_{max}$, the only equilibrium solution sends Customer A to Facility 2, resulting in a total cost of

$d+1$. The price of stability in this case is also $d+1$. $\square$

Looking at the network in Figure 5 another way, for any α_{max} and α_{min} greater than or equal to 0, it is possible to construct a network with price of stability arbitrarily close to $\alpha_{max} - \alpha_{min} + 1$. As we have also seen with the price of anarchy, when the congestion weights of different facilities vary, the decentralized system performance worsens. These results illustrate that even the best equilibrium solutions can be very costly in comparison to optimal decisions made by a central planner. Since centralized decision making is not possible in the settings we consider, alternative methods for improving decentralized decision making are needed.

2.7 Improving Equilibrium Performance

To combat the poor performance described in Sections 2.5 and 2.6, we introduce a way to improve the quality of decentralized equilibrium solutions by influencing the congestion weights associated with facilities. In practice, this may be accomplished by providing information to decentralized decision makers regarding true congestion, changing system processes, or improving waiting time experiences. Congestion weights are now variables to optimized, rather than fixed parameters, and changing congestion weights is a mechanism by which to improve system outcomes. In this section, we begin by establishing the existence of *equilibrium-obtaining* congestion weights. These are congestion weights under which the central optimal solution is also an equilibrium for decentralized decision makers. We characterize the set of such congestion weights and introduce models to identify equilibrium-obtaining α values that optimize policy objectives.

2.7.1 Existence of Equilibrium-obtaining Congestion Weights

The assignment of customers to facilities that optimizes the centralized system objective is computationally easy to identify, but it is not often an equilibrium solution given the current congestion weights associated with the facilities. Given a network, is there a congestion weight vector α for which the central optimal solution is also an equilibrium? To answer this question, we introduce a feasibility problem, P, defined as follows:

$$(P) = \text{Maximize} \quad 0$$

$$x_j^* \alpha_j - (x_k^* + 1)\alpha_k \leq d_{ik} - d_{ij} \qquad \forall\ j, i \in I^*(j), k \neq j$$

$$\alpha_j \geq 0 \qquad \forall\ j$$

Here, $I^*(j)$ is the set of individuals served at facility j in the central optimal solution and $x_j^* = |I^*(j)|$. The objective function is 0 because we are concerned only with finding a feasible α at this point. The constraints require that, for every individual i served at j in the central optimal solution, i's Equilibrium Condition 1 is satisfied with respect to every other facility in the network. The system has a total of $n(m-1)$ constraints, where n is the number of customers and m the number of facilities. Observe, however, that some of the constraints in this system may be redundant. For each facility pair (k, j), there is a constraint for every individual served at j. Among these, the constraint with the smallest right-hand side value, denoted $D_{kj} = min_i\{d_{ik} - d_{ij}\}$, dominates the others. Based on this observation, we can simplify the feasibility problem by eliminating dominated constraints, resulting in problem P':

$$(P') = \text{Maximize} \quad 0$$

$$x_j^* \alpha_j - (x_k^* + 1)\alpha_k \leq D_{kj} \qquad \forall\ j, k \neq j$$

$$\alpha_j \geq 0 \qquad \forall\ j$$

The reduced system has at most $m(m-1)$ constraints. If it has a feasible solution, this means that there exists an α vector under which the central optimal solution is also an equilibrium – an equilibrium-obtaining α. We will show that such a vector always exists, and since the proof relies on duality theory we first present D', the dual of P'. Let y_{kj} be the dual variable associated with the primal constraint $x_j^*\alpha_j - (x_k^* + 1)\alpha_k \leq D_{kj}$. Then D' is as follows:

$$(D') = \text{Minimize} \quad \sum_k \sum_j D_{kj} y_{kj} \tag{5}$$

$$x_j^* \sum_k y_{kj} - (x_j^* + 1) \sum_k y_{jk} \geq 0 \qquad \forall\ j \tag{6}$$

$$y_{kj} \geq 0 \qquad \forall\ k, j \tag{7}$$

P' has a feasible solution if D' is both feasible and bounded. We now prove that this is the case.

Theorem 7 *P' always has a feasible solution α, thus there exists at least one feasible equilibrium-obtaining congestion weight vector.*

We provide the full proof of this result in Appendix A. Clearly the vector $y_{kj} = 0$ for all k, j is a feasible solution to D', and we demonstrate that this is a unique solution.

The result ensures the existence of a feasible α vector. Furthermore, the structure of the problem admits efficient approaches for identifying such a vector. The primal problem is a linear program with two variables per inequality (TVPI). Polynomial algorithms for identifying feasible solutions to TVPI linear programs were presented by [19] and [46]. In the next section, we characterize properties of the feasible region for the problem.

2.7.2 Characterizing the Set of Equilibrium-obtaining α Values

To further understand the properties of the equilibrium-obtaining congestion weight vectors, we now examine the set of equilibrium-obtaining α's by characterizing the maximum and minimum values that components of the vector can assume. This is relevant in practice because it answers questions such as how great the congestion weights *can* be and how great they *must* be while still achieving an equilibrium solution that is optimal. This analysis is facilitated by more generic primal and dual models than those presented in the previous discussion. In what follows, we demonstrate that theoretical properties of these models lead to bounds on the values of equilibrium-obtaining α vectors. These bounds quantify the trade-offs between travel time and congestion necessary to induce an equilibrium solution.

We begin by introducing primal problem $\bar{P}$ and its dual $\bar{D}$. The constraints of $\bar{P}$ are the same as those of problem P', but the objective function is now a generic linear function of α.

$$(\bar{P}) = \text{Maximize} \quad \sum_j b_j \alpha_j \tag{8}$$

$$x_j^* \alpha_j - (x_k^* + 1)\alpha_k \leq D_{kj} \qquad \forall\ j, k \neq j \tag{9}$$

$$\alpha_j \geq 0 \qquad \forall\ j \tag{10}$$

$$(\bar{D}) = \text{Minimize} \quad \sum_k \sum_j D_{kj} y_{kj} \tag{11}$$

$$\lambda_j^* \sum_k y_{kj} - \sum_k y_{jk} \geq \frac{b_j}{x_j^* + 1} \qquad \forall\ j \tag{12}$$

$$y_{kj} \geq 0 \qquad \forall\ k, j \tag{13}$$

The values of objective function coefficients b_j will depend on what we seek to understand about the feasible region of $\bar{P}$. This formulation allows us to examine, for example, the maximum or minimum values that each component α_j can assume.

The dual problem $\bar{D}$, as stated, is a generalized minimum cost flow problem. This problem is similar to the familiar minimum cost flow problem, except flow conservation on arcs is no longer required. Arcs lose or gain flow according to gain factors,

λ. (This framework has been used for modeling currency exchange or the transport of liquid commodities in leaky vessels.) The generalized minimum cost flow problem is also equivalent to a generalized minimum cost circulation problem. Examining the generalized circulation problem formulation leads to important insights about the set of equilibrium-obtaining α values.

To transform $\bar{D}$ into the equivalent generalized circulation problem, we introduce dummy source and sink nodes, S and T, as well as dummy arcs (S, j) and (j, T) for each original node j, (T, S) to connect the sink to the source, and a self-loop (S, S). Each dummy arc has a cost of 0 and an associated gain factor of 1, with the exception of (S, S) with $\lambda_{SS} > 1$ to ensure total flow balance.

$$(\hat{D}) = \text{Minimize} \quad \sum_k \sum_j D_{kj} y_{kj} \tag{14}$$

$$\lambda_j^* \sum_k y_{kj} - \sum_k y_{jk} + y_{Sj} - y_{jT} = 0 \qquad \forall\ j \tag{15}$$

$$y_{TS} + \lambda_{SS} y_{SS} - \sum_j y_{Sj} - y_{S,S} = 0 \tag{16}$$

$$\sum_j y_{jT} - y_{T,S} = 0 \tag{17}$$

$$y_{kj} \geq 0 \qquad \forall\ k, j \tag{18}$$

$$y_{Sj} = 0 \qquad \text{for } \{j \mid \frac{b_j}{x_j^* + 1} \geq 0\} \tag{19}$$

$$0 \leq y_{Sj} \leq \frac{b_j}{x_j^* + 1} \qquad \text{for } \{j \mid \frac{b_j}{x_j^* + 1} < 0\} \tag{20}$$

$$y_{jT} \geq 0 \qquad \text{for } \{j \mid \frac{b_j}{x_j^* + 1} \leq 0\} \tag{21}$$

$$y_{jT} \geq \frac{b_j}{x_j^* + 1} \qquad \text{for } \{j \mid \frac{b_j}{x_j^* + 1} > 0\} \tag{22}$$

In the generalized circulation problem, supplies and demands are no longer associated with the nodes. All flow is endogenous and circulates within the network, so the flow balance at each node is 0 as represented in constraints (15)-(17). The transformation from the original generalized minimum cost flow problem to the generalized minimum cost circulation problem replaces node supplies from the original dual constraints (12) with flow bounds on appropriate arcs, given by constraints (19)-(22).

We use this transformed dual problem to establish several characteristics of the feasible region of equilibrium-obtaining α values in problem P'. First, we demonstrate that equilibrium-obtaining α values can be infinitely large.

Theorem 8 *If all non-zero primal objective function coefficients b_j are positive, the primal problem is unbounded.*

Proof. Let J^+ be the set of facilities for which the primal objective function coefficients b_j are non-zero. From the statement of the theorem, $b_j > 0$ for all $j \in J^+$. Then $y_{Sj} = 0$ for all j to satisfy the constraint set (19). However, we must also have $y_{jT} \geq \frac{b_j}{x_j^*+1} > 0$ for all $j \in J^+$. This is a contradiction, because it is impossible to have flow exiting the original network at the sink T without flow entering at the source S. The dual problem is thus infeasible. Since the primal problem always has a feasible solution (Theorem 7), the primal problem is unbounded. □

We have seen that equilibrium-obtaining α values *can* be infinitely large. How large *must* they be? That is, what is the minimum importance that decentralized decision makers must place on congestion for a particular facility to ensure that the central optimal solution is an equilibrium? Suppose we want find the minimum value that α_j can take on for some facility j. That is, the primal objective is to maximize $-\alpha_j$. Duality theory ensures that we can determine the minimum value of α_j by solving the generalized minimum cost circulation problem $\hat{D}$ in the dual

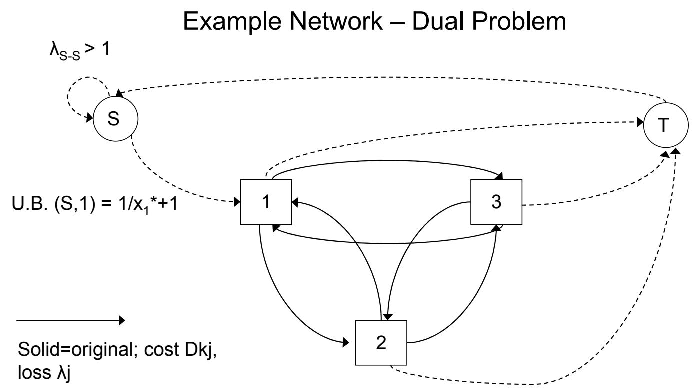


Figure 6: Example Dual Network for Generalized Circulation.

network. However, we prove that there is a simpler way to find the solution. Figure 6 illustrates the dual network for a case in which we seek to minimize α_1 in a problem with three facilities.

Theorem 9 *The minimum feasible value of α_j for any j can be found by solving a single generalized shortest path problem in the dual network.*

Proof. First, we show that it suffices to consider S-T flow rather than the entire circulation. This is evident because the costs of the (T,S) dummy arc and (S,S) dummy loop are each 0 and the (S,S) dummy loop is constructed only to restore flow balance, if needed, in the original circulation problem. Therefore, flow on these arcs is completely dictated by the flow between S and T and will not affect the value of the minimum cost circulation.

Next, we must show that a min cost S-T flow can be achieved using a single simple path. For any S-T path p in the network, the unit cost of flow on p is given by:

$$C_p = \sum_{(k,j)\in p} \left(D_{kj} \cdot \prod_{l \leq k, l \neq 1} \lambda_l \right),$$

where $(k, j) \in p$ are the arcs in path p, $l \leq k$ indicates nodes l appearing in p prior to and including k, D_{kj} is the cost of arc (k, j), and λ_l is the loss factor associated with arcs entering node l.

Now, if $C_p \geq 0$ for all p that contain at least one original network arc, then clearly a minimum cost flow is given by the 0 flow or by an S-1-T-S flow, either of which has value 0, so there is nothing to show in this case.

If at least one $C_p < 0$, then the minimum cost flow can be found by successively saturating the remaining path with the most negative cost. However, since all paths share the common arc (S,1) with upper bound $\frac{1}{x_1^*+1}$, and no other arcs have upper bounds, only the path with most negative cost will be selected. The value of the flow is then given by $\frac{1}{x_1^*+1} \cdot C_{p_{min}}$, where p_{min} is the path with most negative cost.

Since the value of the primal optimal solution is equal to the that of the dual optimal solution, and we have just shown that the dual optimal value can be obtained by finding the S-T path with the most negative cost, we can determine the minimum value of α_1 by solving the minimum cost S-T path problem, as proposed. □

Solving the shortest path problem will therefore answer the question about the minimum congestion weight that decentralized decision makers must place on a particular facility. This approach also lends itself to the development of bounds on the values of individual α_j components. To facilitate further characterization of these bounds, we introduce a technical lemma; the proof is presented in Appendix A.

Lemma 4 *Although negative cycles consisting exclusively of original nodes and arcs in the dual network may exist when flow loss factors are included, such a negative cycle is not included in any most-negative $S-T$ path in the dual network.*

This result provides information about the structure of this class of generalized minimum cost circulation problems. In our case, it will be used to establish bounds characterizing the minimum value that each component of α can assume, or in other words, the minimum level of importance that decentralized decision makers must place on facility congestion for the central optimal solution to be an equilibrium. Bounding the smallest equilibrium-obtaining values of congestion weights provides insights for managing decentralized systems by identifying trade-offs between travel time and congestion even without solving the shortest path problem for each value of α_j. This can point to opportunities to provide information or change system processes in ways that influence individuals' congestion weights, thereby improving decentralized decision making.

Theorem 10 *The minimum feasible value of α_j for any j is bounded by this expression:*

$$\alpha_j =\leq -\frac{1}{x_j^* + 1} D_{min} \left(\frac{1 - \lambda_{max}^{m-1}}{1 - \lambda_{max}} \right),$$

where $D_{min} = min_{k,j}\{D_{kj}\}$ and $\lambda_{max} = max_j\{\frac{x_j^*}{x_j^*+1}\}$. Moreover, this bound is tight.

The proof of this result is presented in Appendix A. Intuitively, the bound captures the trade-off between travel time and congestion necessary to induce an equilibrium solution. In the bound expression, the values D_{min} and λ_{max} capture the arc with least cost and the arc on which there is the smallest loss of flow, respectively. It is instructive to examine what happens to the value of the bound when λ_{max} approaches its limiting values. Recall that $0.5 \leq \lambda_{max} < 1$. When $\lambda_{max} = 0.5$, no more than one individual is served at each facility and $lim_{\lambda_{max} \to 1/2} \left(\frac{1-\lambda_{max}^{m-1}}{1-\lambda_{max}} \right) = 2 - (1/2)^{m-2}$. The shortest path bound is dominated by the loss of flow that occurs along the path. On the other hand, the value of λ_{max} approaches 1 when there is a facility that is very congested. Considering again the last term of the shortest path bound, we have $lim_{\lambda_{max} \to 1} \left(\frac{1-\lambda_{max}^{m-1}}{1-\lambda_{max}} \right) = m-1$, where m is the number of facilities. In other words, the shortest path is bounded by $m-1$ times the cost of the most negative arc and there is very little loss of flow. Thus, when the central optimal solution is more congested, greater congestion weights are required to ensure that this solution is an equilibrium. This bound is tighter in systems where many λ_j values are close to 1, that is, systems in which many facilities serve lots of customers.

Using similar ideas, we also obtain a path-based bound on the sum of all α_j values.

It is also useful to characterize the minimum sum of α_j values, especially when there is a cost associated with influencing the importance that decentralized decision makers place on congestion. The proof of this result is presented in Appendix A.

Theorem 11 *The minimum feasible value of $\sum_j \alpha_j$ is bounded by this expression:*

$$\sum_j \alpha_j \leq -\frac{x^*_{max}+1}{x^*_{min}+1} \cdot D_{min}\left(m - \left(\frac{1-\lambda^m_{max}}{1-\lambda_{max}}\right)\right)$$

Moreover, this bound is tight.

Again, the congestion weights implied by this bound capture the trade-off between travel time and congestion that is necessary for the central optimal solution to be an equilibrium under decentralized decision making.

2.7.3 Optimizing α

Although it is useful to know that feasible α vectors exist under which the central optimal solution is an equilibrium, more useful in practice is the ability to identify such congestion weights that also optimize particular policy objectives. In this section, we introduce three such objectives and demonstrate that finding optimal α values in these scenarios can be done efficiently.

Before doing so, recall that the dual of problem $\bar{P}$ is equivalent to a generalized circulation problem. Wayne [102] developed a polynomial algorithm for solving these problems that relies on their underlying combinatorial structure. It can be used to optimize $\bar{P}$ under any linear objective. As we will demonstrate, important planning goals can be incorporated into this problem structure. We begin by presenting three variants of $\bar{P}$, after which we summarize observations about these problems.

The first objective we consider is that of **minimizing the range of congestion weights** in the system while still finding an α vector under which the central optimal solution is an equilibrium. In the context of managing a system of public health or emergency response facilities, this objective represents a desire to reduce perceived discrepancies between facilities. Problem $\bar{P}_R$ is a modification of our primal problem $\bar{P}$ that captures this goal. Although we want to minimize the difference between α_{max} and α_{min}, we maintain the maximization objective for consistency with the earlier presentation.

$$(\bar{P}_R) = \text{Maximize} \quad -(\alpha_{max} - \alpha_{min}) \tag{23}$$

$$x_j^* \alpha_j - (x_k^* + 1)\alpha_k \leq D_{kj} \qquad \forall\ j, k \neq j \tag{24}$$

$$\alpha_{max} - \alpha_j \geq 0 \qquad \forall\ j \tag{25}$$

$$\alpha_j - \alpha_{min} \geq 0 \qquad \forall\ j \tag{26}$$

$$\alpha_j \geq 0 \qquad \forall\ j \tag{27}$$

Here, constraint set (24) comes directly from feasibility problem P'. Constraints (25) and (26) define the variables α_{max} and α_{min} in terms of the α_j's.

System planners may also be concerned with the total or maximum change in congestion weights that must be made across facilities to achieve an equilibrium solution. **Minimizing the total change in congestion weights** can be viewed as minimizing the required budget for change, since affecting the importance decision makers place on congestion requires investments in information dissemination or changes in facility operations. Problem variant $\bar{P}_{sum-\Delta}$ seeks to minimize the total change in congestion weights across all facilities necessary to reach an equilibrium-obtaining α vector. In this context, we let α_j^0 denote the original congestion weight associated with facility j. Variables δ_j^+ and δ_j^- measure the positive or negative change, respectively, between α_j^0 and the candidate congestion weights α_j. Again, we frame $\bar{P}_{sum-\Delta}$ as a maximization problem for consistency.

$$(\bar{P}_{sum-\Delta}) = \text{Maximize} \quad -\sum_j \delta_j^+ + \delta_j^-) \tag{28}$$

$$x_j^*\alpha_j - (x_k^* + 1)\alpha_k \leq D_{kj} \qquad \forall j, k \neq j \tag{29}$$

$$\alpha_j \leq \alpha_j^0 + \delta_j^+ \qquad \forall j \tag{30}$$

$$\alpha_j \geq \alpha_j^0 - \delta_j^- \qquad \forall j \tag{31}$$

$$\alpha_j \geq 0 \qquad \forall j \tag{32}$$

$$\delta_j^+ \geq 0 \qquad \forall j \tag{33}$$

$$\delta_j^- \geq 0 \qquad \forall j \tag{34}$$

Constraint set (29) comes directly from P', while constraints (30) and (31) define the variables α_j in terms of the original congestion weight α_j^0 and the weight change δ_j^+ and δ_j^-, respectively.

Similarly, the goal of problem variant $\bar{P}_{max-\Delta}$ is **minimizing the maximum change in congestion weights**. This objective is relevant if planners want to spread their resources across the system and avoid solutions which require large changes in the weights of only one or a few facilities. We measure the maximum change in congestion weights using δ_{max}. The problem of minimizing the maximum change in perception is defined as:

$$(P'_{max-\Delta}) = \text{Maximize} \quad -\delta_{max} \tag{35}$$

$$x_j^*\alpha_j - (x_k^* + 1)\alpha_k \leq D_{kj} \qquad \forall j, k \neq j \tag{36}$$

$$\alpha_j \leq \alpha_j^0 + \delta_j^+ \qquad \forall j \tag{37}$$

$$\alpha_j \geq \alpha_j^0 - \delta_j^- \qquad \forall j \tag{38}$$

$$\delta_{max} \geq \delta_j^+ \qquad \forall j \tag{39}$$

$$\delta_{max} \geq \delta_j^- \qquad \forall j \tag{40}$$

$$\alpha_j \geq 0 \qquad \forall j \tag{41}$$

$$\delta_j^+ \geq 0 \qquad \forall j \tag{42}$$

$$\delta_j^- \geq 0 \qquad \forall j \tag{43}$$

Constraints (36), (37), and (38) are analogous to the first three constraints of $P'_{sum-\Delta}$. Constraints (39) and (40) require the new variable δ_{max} to be at least as large as each of the δ_j^+ and δ_j^- values.

These three variants of $\bar{P}$ optimize important policy objectives, and their structure admits an efficient algorithm for finding solutions.

Remark 5 *Problems $\bar{P}_R$, $\bar{P}_{sum-\Delta}$, and $\bar{P}_{max-\Delta}$ are each TVPI linear programming problems. Because the additional constraints required for these formulations (namely, (25) and (26) in $\bar{P}_R$, (30) and (31) in $\bar{P}_{sum-\Delta}$, and (37), (38), (39), and (40) in $\bar{P}_{max-\Delta}$) are simply definitional, feasibility for each problem follows from Theorem 7. Moreover, Wayne's algorithm [102] can find an optimal solution in polynomial time.*

This observation assures that we can not only find an α vector under which the central optimal solution is an equilibrium, but also that this can be done while minimizing either the range of congestion weights or the total or maximum change in weights. The ability to do this efficiently is an important step toward informing decentralized system design.

2.8 Conclusions

This work poses several questions regarding the design and analysis of decentralized logistics systems, with a particular focus on problems motivated by humanitarian response. Decentralized decision making is common in such settings, and the use of optimization approaches that adopt a centralized perspective can lead to poor system performance. The work presented here integrates tools from optimization and game theory to develop prescriptive models for logistics networks that account for decentralized behavior. It also makes fundamental advances in the design and management of networks with selfish users. We present an efficient algorithm for finding stable decentralized solutions and find new price of anarchy and price of stability results for a class of decentralized network problems, and demonstrate that system performance varies depending on the weights that decentralized decision makers place on congestion. We introduce a novel mechanism for improving decentralized performance by changing congestion weights and explore insights about what these values can or must be. In Chapter 3, we demonstrate that these advancements are useful in practice with an application using data from a large-scale distribution scenario.

The answers to the questions posed here will add to the fundamental understanding of decentralized systems and provide tools for their management. They will also provide a base upon which researchers can build further, including the incorporation of congestion weights that are specific to individuals in addition to those associated with facilities, learning on the network, social interactions among agents, and the dynamics of decentralized systems over time. As such, they represent an important contribution to the theory and practice of managing decentralized logistics systems.

CHAPTER III

THE IMPACT OF DECENTRALIZED DECISIONS ON THE EQUITY OF SUPPLY DISTRIBUTION

3.1 Introduction

In this chapter, we examine the impact of decentralized decision making using data from an actual large-scale distribution effort. This study makes several important contributions. First, it demonstrates that the methods introduced in the previous chapter are applicable to large-scale problems arising in practice. It integrates recent developments in the field of spatial statistics with optimization, game theory, and geographic information systems (GIS). This novel integration makes use of data from many sources to explain the impact of past decisions about distribution system design, which in turn can lead to improved decisions in the future. The results confirm the importance of accounting for decentralized behavior in system design and point to opportunities to use the mechanism from the first part of the thesis in future distribution efforts of this nature.

3.2 Background

This study is based on an effort to deliver a particular type of product to a large number of people in a limited time period during which demand for the product exceeded supply. The work is done in collaboration with the organization responsible for coordinating the effort, but details of the project are withheld until authorization has been given for material to be made public. Where complete citations of data sources cannot provided in the context of this thesis document, full details will be made available upon approval for release in a published paper describing this work to the scientific community and policy makers. In this chapter, we will simply refer to national, state, and local decision makers, with products distributed directly to consumers from service locations.

The supply chain for the product consisted of several stages. The national organization procured the product from manufacturers, who sent the product via third-party distribution centers to service locations designated by each state. (In some regions, the supply chain included a stage of local distribution. There was limited visibility of this stage, so it is excluded from our analysis.) Individuals chose where to pick up the product from among the service locations. In the stage of the distribution that we study, demand for the product significantly outpaced supply. The centralized organization allocated the limited supply among state partners using a pro rata method, meaning that each state received a fraction of the total supply equal to that state's proportion of the total population. Each state then determined how its allocation would be divided among the designated service locations, each of which had to submit requests for the product. Since requests exceeded availability, some states adopted a pro rata policy similar to the national organization while others prioritized certain types of service locations or used other methods to allocate the product. While the national pro rata approach represents an equal allocation from the state point of view, it does not preclude differences in individuals' access to the product. Our study examines the shipments of product at the service location level and the resulting availability at the census tract level to identify potential inequities arising from the distribution approach described.

To identify inequities, if they exist, we model the distribution system from two perspectives. The first uses optimization and game theory to represent the choices of individuals in the system, who choose from among facilities depending on distance, the number of people at a facility, and the quantity of product available. We contrast the results of this model, which incorporates individuals' choices, with a traditional optimization model that assumes a centralized planner can control actions across the entire system to minimize the total distance and facility congestion. The output of the optimization models allows us to measure individuals' access to the product. We then use spatial statistics to examine differences in accessibility across space and correlations between accessibility and socioeconomic variables.

3.3 Research Goals

The research goals of this study are three-fold. First, we model an actual product distribution scenario using the ideas presented in Chapter 2 and examine the differences in system outcomes under the centralized and decentralized frameworks. Second, we measure individuals' access to the product as a function of the distance traveled and congestion experienced in obtaining the product and examine inequities in accessibility across the system. Finally, we explain the observed inequities as functions of spatially-correlated variables such as demographic characteristics and the availability of related service locations. Together, these goals lead to insights that can inform future response policies and planning.

The remainder of this chapter is organized as follows. In Section 3.4 and 3.5, we discuss the data sources and methods used in this study, respectively. Section 3.6 notes the limitations of the study. We present our results in Section 3.7 and discuss interpretations and recommendations in Section 3.8. We conclude in Section 3.9 with a description of ongoing and future research related to this study.

3.4 Data

This study makes use of three types of data, namely those that characterize product demand, product supply, and socioeconomic factors that may be correlated with modeling outcomes.

Product Demand: We take the entire population of the affected area as the demand for the product in our study, and we examine populations at the census tract level. According to the Census Bureau [94], census tracts are "small, relatively permanent geographic entities within counties." They are designed to have between 2500 and 8000 residents, with an average population of about 4000. Census tracts are determined with community input and are relatively homogenous with respect to economic status and demographics, at least at the time that they are established. We choose the census tract level because it allows us to look at local impacts of decisions made at the state or national level. We assume that the population of a census tract is located at its center of population, also called the population centroid [96]. We use the latitude and longitude coordinates of this centroid to calculate distances to the facilities that are providing the product. The distances are determined from each census tract to every service location within 50 miles using the U.S. highway network [77] via GIS software [32].

Product Supply: The street addresses of about 50,000 service locations and the quantities of product available at each location during the shortage period were provided by the national organization. We geocode the addresses of these locations, which requires using GIS software [32] to match street addresses to latitude and longitude coordinates. The geocoding success rate is 90 percent for the service locations nationwide. We impute coordinates for an additional 8.6 percent of the locations by sampling a random point within the zip code associated each location.

Socioeconomic Factors: In our statistical analysis, we examine a number of socioeconomic factors to determine their relationships with the model outcomes. We obtain data on per capita income, median household income, and race/ethnicity from Community Sourcebook·America – ESRI [31]. These data are acquired at the census tract level and updated yearly based on U.S. Census Bureau data. Using the area of each census tract [95], we calculate its population density. The percent of census tract population living below the federal poverty level is obtained from The Public Health Disparities Geocoding Project [90]. Finally, we obtain a listing of all locations that could potentially distribute the product from a national database and geocode their addresses. (To ensure confidentiality of the work, this source is withheld from the thesis.) We calculate the ratio between the number of potential service locations within 20 miles of each census tract and the tract's population density. This value quantifies the relative availability of possible service locations, and we explore this variable to determine the degree to which larger service infrastructure issues contribute to differences in accessibility to the product in question.

3.5 Definitions and Methods

The methods we use to measure the accessibility of the product and understand differences in accessibility associated with socioeconomic variables are described in this section. We begin by defining accessibility and equity in the context of this study, introduce the models we use to obtain measures of accessibility, and describe statistical procedures we use to identify factors associated with inequities.

3.5.1 Measuring Accessibility and Equity

The literature contains many definitions of both accessibility and equity in access to public services [11, 23, 54, 62, 65, 66, 85, 86]. As in [54], we make the distinction between potential and realized access. Potential access is a measure of the availability of the service to an individual and is most often characterized by distance. Realized access indicates whether or not an individual actually received the service and depends upon many factors, including potential access and demographic variables. Data concerning realized access are not available at the local level for the majority of our study area. Consequently, we develop models of the distribution system to measure individuals' potential access to the product. **Potential access** in the context of this study is the availability of the product to an individual and depends on the distances to facilities distributing the product, the number of other people who choose a facility, and the quantity of product available at each location.

We define three metrics to quantify potential access, the values for which are calculated using the output of the optimization models described in Section 3.5.2. First, we compute a distance metric for each census tract by averaging the distances traveled by each of the individuals in that census tract to produce an aggregate measure. Many models that examine decisions affecting access to public goods and services use distance as a principle component of individuals' utility functions, including gravity-based models that are common in designing and assessing public service systems (for example, [45, 54, 62]), location models for community health centers [40], and general facility location models [26]. Our second metric is the average congestion (or viewed another way, the scarcity) experienced by people from a census tract, where congestion at a facility is defined as the number of people per product available at that facility. Congestion is a key component of utility in traffic models for both highway and computer networks (for example, [82]) and scarcity has been shown to dissuade individuals from seeking products in some contexts [15]. Finally, we consider the sum of the first two metrics. Utility functions that combine a constant component, such as distance, with a congestion component have been widely used in computer science and traffic theory to incorporate customer utility. Moreover, researchers have identified the need for studies that integrate congestion and distance measures in location modeling [30, 78]. Together, the distance, congestion, and combined metrics allow us to examine the different components of potential access.

We seek to understand inequities in potential access as they relate to socioeconomic factors and other system variables. For a summary of different measures of equity, the reader is referred to the review by Marsh and Schilling [66]. Operations research and management science have traditionally been concerned with efficiency and effectiveness rather than equity, but in particular there are a number of facility location models that include some notion of equity [30, 65, 71, 85]. These papers are meant to be an illustrative, not exhaustive, list; for a thorough review of equity in location modeling, see [30] and the references cited therein. Braveman and Gruskin [11] address the reasons that studying equity is important, stating: "Health inequities put disadvantaged groups at further disadvantage with respect to health, diminishing opportunities to be healthy." While their work focuses on health, similar reasons apply to equitable access to food, clean water, and other necessities. These reasons are relevant to our study, because an inadequate distribution system can lead to poorer outcomes for those affected by this scenario. Increasingly, researchers are using GIS tools and census information to examine health outcomes and equity in access to public services as a function of socioeconomic variables; see, for example, [13, 58, 88].

We adopt a definition of equity that is similar to that described in [11]. **Equity** in potential access to the product is the absence of systematic disparities in access between different groups of people, identified by location or underlying socioeconomic variables. In the context of the problem we consider, understanding differences in potential access that are associated with identifiable social groups or characteristics can inform policy decisions. For example, this may help determine how much product to send to which locations or whether it is necessary to recruit additional service locations in specific regions. Using this information, future response efforts can focus on plans, interventions, or resource allocations that have greater impact and improve equity for all those affected.

3.5.2 Optimization Models

We adopt two modeling approaches to evaluate the potential accessibility of the product in this study and compare their outputs. Before introducing the models themselves, we discuss the merit of using optimization to determine the accessibility of this product. Other alternatives might be measuring the number of products available per census tract or per person in a census tract, counting all census tracts within a certain number of miles from a facility as being served by that location, or counting all facilities within a threshold distance as serving a particular census tract. These alternatives suffer from one or more drawbacks. The first imposes artificial boundaries on individuals' willingness to travel to receive the product. The second leads to double-counting of *people* since populations may be close to multiple facilities; this does not provide an accurate picture of the access to the product. Conversely, the third leads to double-counting of *products*. None of these approaches accounts for individuals' decreasing willingness to travel as distances increase nor the interaction between distance traveled, number of people (congestion) at a facility, and the number of available products. More sophisticated gravity-based models (for example, [45, 54, 62]) account for these factors, but these approaches do not model actual choices about which facility to visit and the resulting outcomes of those choices. Optimization is well-suited to handle these aspects.

The optimization models we use to determine the assignment of individuals to facilities are adapted from those presented in Chapter 2. The reader is referred to that part of the thesis for a detailed discussion of the models and their theoretical properties. Here we present the specific implementations used in this study. We examine the challenge of allocating the product from two perspectives: that of individuals choosing where to pick up the product in a way that optimizes their own objectives, and that of a centralized planner with the ability to assign each person in the system to optimize overall system efficiency. The input for both models includes the shipment locations, product quantities received during the shortage period, census tract populations, and distances between census tract centers of population and facilities. Each model produces metrics, namely, the average distance, average congestion, and sum of these values, that we then use to evaluate the potential accessibility of the product at the census tract level.

In contrast to the models presented in Chapter 2, both the decentralized and centralized models now account for the **capacity** of each facility. We define capacity as the number of products available at the facility. Note that despite this addition to the modeling framework, the results on the complexity of finding a stable equilibrium solution (1) and on finding an equilibrium-obtaining congestion weight vector (7) still hold.

3.5.2.1 Decentralized Model

Individuals' choices about which facility they will visit to receive the product are a key component of our models. We assume that individuals know where the product is available and in what quantities. (In this and many similar scenarios, such information is often available online.) Given these parameters, we model user choice as a network congestion game with unweighted, atomic, unsplittable flow (defined in Chapter 2). For model tractability, we assume that each census tract population is divided into communities of size c and therefore round the population of each census tract to the nearest number. (For this study, $c = 100$.) All c individuals in a community are assumed to visit the same facility, but different communities within a census tract may visit different facilities. We seek an equilibrium, or stable, decentralized solution, which is defined as a solution that satisfies the following condition for all communities.

Definition 2 *Equilibrium Condition for Community i:*

$$d_{ij} + \frac{1}{s_j} \sum_{l=1}^{n} cx_{lj} \leq d_{ik} + \frac{1}{s_k} \left(\sum_{l=1}^{n} cx_{lk} + c \right) \quad \forall\ k \neq j. \tag{44}$$

Here j is community i's facility in the solution being considered, while $x_{lj} = 1$ if community l is served at facility j and 0 otherwise. Values d_{ij} and d_{ik} represent the distance from community i to facilities j and k, respectively. The community size is given by c. The value s_j is the product supply available at facility j.

Expression (44) compares the cost incurred by an individual in community i if facility j is chosen with the cost of an alternative facility k. If the inequality holds for all alternative facilities, then community i has no incentive to switch unilaterally.

The results of Theorem 1 in Chapter 2, which states that an equilibrium solution can be found efficiently using a minimum cost flow algorithm on a transformation of the decentralized network, still hold for this problem. Therefore, we use the minimum cost flow approach previously developed to find a solution to the decentralized problem in this setting.

3.5.2.2 Centralized Model

For comparison to the decentralized model, we also examine a centralized planner's model. This modeling approach assumes that a single decision maker can determine where all individuals will obtain the product. The planner's goal is to minimize the total travel distance and congestion while ensuring that all product is distributed. This approach is similar to the centralized model presented in Chapter 2, but it has two key differences. First, the centralized planner's notion of congestion now depends not only on the number of individuals served but also on facility capacity, or the number of items available. Second, we require that all of these items be distributed. Recall that this study is concerned with the distribution phase during which demand is greater than supply, so the centralized planner seeks to have the greatest impact by making use of this limited supply. The optimization model from the centralized perspective is thus:

$$\text{Minimize} \quad \sum_{i=1}^{n}\sum_{j=1}^{m} d_{ij}p_i x_{ij} + \sum_{j=1}^{m} \frac{1}{s_j}\left(\sum_{i=1}^{n} p_i x_{ij}\right)^2 \tag{45}$$

$$\sum_{j=1}^{m} x_{ij} = 1 \qquad \forall i \tag{46}$$

$$\sum_{i=1}^{n} p_i x_{ij} \geq s_j \qquad \forall j \tag{47}$$

$$0 \leq x_{ij} \leq 1 \qquad \forall i, j \tag{48}$$

In the centralized planner's model, decision variable $x_{ij} \in [0, 1]$ denotes the fraction of the population of census tract i ($i = 1, \ldots, n$) served at facility j ($j = 1, \ldots, m$). The parameters of the model include p_i, the population of census tract i; d_{ij}, the travel distance from census tract i to facility j; and s_j, the supply of products shipped to facility j. Constraint (46) requires that the entire population of each census tract be assigned to a facility, while constraint (47) enforces the requirement that all of the product be distributed.

3.5.3 Statistical Analysis of Results

The optimization models just described produce distance and congestion metrics at the census tract level. These are measures of the access that individuals in that census tract have to the product. We seek to understand how and to what extent accessibility is correlated with the socioeconomic characteristics of the census tract population and with the availability of potential service locations. To examine these relationships, we conduct a statistical analysis.

Both the output of the optimization models (distance and congestion metrics) and the socioeconomic factors are spatially correlated. It is necessary to account for this correlation in the statistical procedures used to estimate the associations between factors and response. To accomplish this, we implement a statistical method called a varying coefficient model. Like traditional linear regression, this approach estimates coefficients for the factors and fits an expression to predict the response variable. However, unlike linear regression, the regression coefficients are permitted to vary across geographic space or time. Researchers have applied varying coefficient models to time-dependent data [34, 43, 47, 49, 106, 108], data that vary spatially [6, 38, 101], and data that vary both in time and space [86]. Gelfand et al. [38] and Waller et al. [101] review existing varying coefficient models applied to spatial data and propose Bayesian procedures to model these scenarios. A Bayesian procedure can account for multi-collinearity in the explanatory variables, but such methods are computationally intensive even for small data sets. Since our data consists of a large number of geographic locations, we estimate the space-varying coefficients using recently-developed nonparametric methods.

We introduce the following space-varying coefficient model to estimate the relationships between accessibility of the product and the socioeconomic factors.

$$\mathbb{E}[Y_j|X] = \beta_0(g_j) + \beta_1(g_j)X_{1,j} + \ldots + \beta_R(g_j)X_{R,j} \tag{49}$$

In this model, the observed data are denoted $(Y_j, \{X_{r,j}, r = 1, \ldots, R\},)$ where $Y_j = Y(g_j)$ is the response variable (either distance or congestion) and $X_{r,j} = X_r(g_j)$ is a set of covariates (socioeconomic factors) observed at location $g_j = (g_{j1}, g_{j2})$, for $j = 1, \ldots, G$. In our model, g_{j1} and g_{j2} simply denote the latitude and longitude of the census tract center of population. Finally, $\beta_r(g)$ for $r = 1, \ldots, R$ are smooth coefficient functions that may vary in space. The first term of the expression, $\beta_0(g_j)$, represents a space-varying intercept value.

We estimate the unknown coefficient functions, $\beta_r(g)$ for $r = 1, \ldots, R$, using nonparametric methods, namely thin plate splines [104]. This approach is selected because it balances goodness of fit, smoothness of resulting coefficient functions, and computational effort. It is implemented using functions in the R statistical software library *mgcv* [104]. The output of the procedure includes both estimates of the coefficient functions themselves and information to make inference on the shapes of these functions. Not all regression coefficients exhibit statistically significant variability in space. We use simultaneous confidence bands, introduced in [86], to determine whether coefficients are constant, linear, or nonlinear across space. Based on these results, we also examine the statistical significance of constant coefficients using conventional hypothesis testing. Using this approach, we determine the set of socioeconomic factors that best explains the spatial variability in access to the product.

3.6 Limitations

This study has several limitations. It assumes that, although decentralized decision makers may consider several locations and the number of people likely to visit each one, they ultimately choose only one location for service. For model tractability in the decentralized scenario, we assume that all members of communities of a specified size choose the same location. Census tracts whose total population is less than half of the community size are ignored by that model. (This represents less than 0.0002 percent of the population for the regions discussed in this chapter.) The decentralized model identifies one equilibrium solution, but others may exist. Distances in our models were measured from the center of population of the census tract to the service location using the U.S. highway network. These distances are likely to be less accurate for census tracts with large areas and widely dispersed populations, as well as for areas with limited U.S. highway infrastructure. In the accessibility metric that combines both distance and congestion, we assume that these values can be represented on a common scale (for example, units of time or disutility) without further conversion; other methods of scaling the values for compatibility could be employed. Geocoded locations could not be identified for 1.7 percent of service locations nationwide; these are excluded from the study. This analysis considers only the product quantities during the shortage period. It takes all product shipments during that period as a single set and ignores redistribution of the product beyond the service locations selected by the state partners; the latter is more appropriate in some states than in others. In the statistical analysis, we have used data from a national database of potential service locations relevant to this distribution effort to estimate the availability of such services. To our knowledge, actual data regarding receipt of the product by individuals at the local level is not available for most regions.

3.7 Results

Here we present the results of the study for the state of Georgia. The analysis includes 1,616 census tracts with a total population of 8,186,453, as well as over 2,000 service locations representing a total stock of more than 2,000,000 items during the shortage period. In Section 3.7.1, we compare the allocation of products under the centralized and decentralized optimization models. This is followed by a statistical analysis of the relationships between the congestion accessibility metric and a set of socioeconomic factors in Section 3.7.2.

3.7.1 Allocations and Accessibility Measures

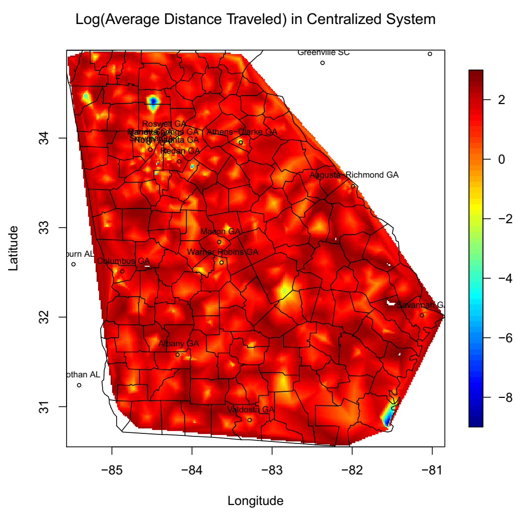
Both the centralized and decentralized optimization models introduced in Section 3.5.2 were solved using CPLEX 12.1 optimization software [51] implemented on a Debian 4.0/4.1 GNU/Linux x86-64 system with access to 32 GB RAM. A community size of 100 people was used in the decentralized model. (One census tract with a population of only 18 was thus excluded from that model.) An equilibrium solution to the decentralized model was found in 5.03 hours, while the centralized model required 1.00 hour.

In Figures 7-9, the resulting distance, congestion, and total metrics for the centralized (a) and decentralized (b) models are illustrated, along with the difference in metrics between the two models (c). The figures plot the natural log of the respective metrics measured at the census tract level and smooth these values over space to produce a continuous picture of the variability in potential spatial accessibility. In Figure 7, the red part of the color spectrum indicates larger distances traveled for service and thus lower accessibility, while blue colors indicate smaller distances and thus greater accessibility. In Figure 8, red (respectively, blue) indicates larger (smaller) numbers of people relative to the available product quantity. Similarly in

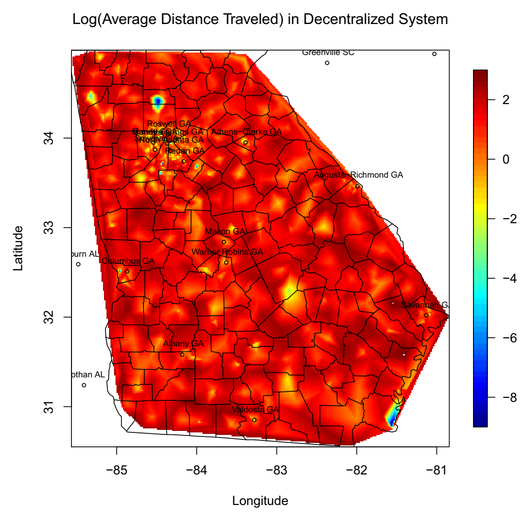
Figure 9, the red part of the spectrum identifies areas with larger total costs, whereas areas shown with blue experienced smaller totals of distance and congestion.

In Figure 7, the differences between distances traveled in the centralized and decentralized systems are subtle, but examining the figures together illustrates that there are generally fewer and smaller areas of dark red in the decentralized map. This indicates that distances were somewhat smaller in the decentralized system, which occurs because communities are optimizing local objectives. Figure 7(c) plots decentralized minus centralized distance, further clarifying the areas where the two results differ most. Across census tracts, distances traveled in the centralized system ranged from 0 to 24.8 miles (recall that the map has a logarithmic scale), while those in the decentralized system were between 0 and 21.5 miles. Although the centralized planner may impose greater travel to mitigate congestion, the decentralized decision makers do not. Note also that in both cases, there are a few locations with extremely small travel distances. Areas with larger distances tend to be more rural in both cases, although not exclusively.

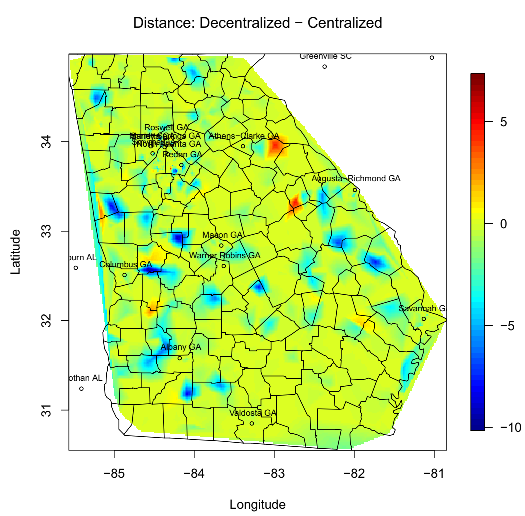
Figure 8 illustrates the resulting congestion relative to facility capacity for the centralized and decentralized models, as well as the differences between the two systems. The values are between 1.0 and 19.0 persons per product in the centralized system, while the decentralized congestion ranged from 0.5 to 25.4 persons per product. The colors in the centralized map on the left are more homogeneous than those in the decentralized map on the right. This indicates that the decentralized solution shows more variation in congestion across space; the congested areas are more congested, while the areas with excess vaccine are more definitive as well. Here, the darkest blue colors represent negative natural log values and indicate that the number of products actually exceeds the number of individuals served in that area. When the community size is 100, approximately 1.1 percent of facilities had some unallocated product; this



(a) Centralized Distance

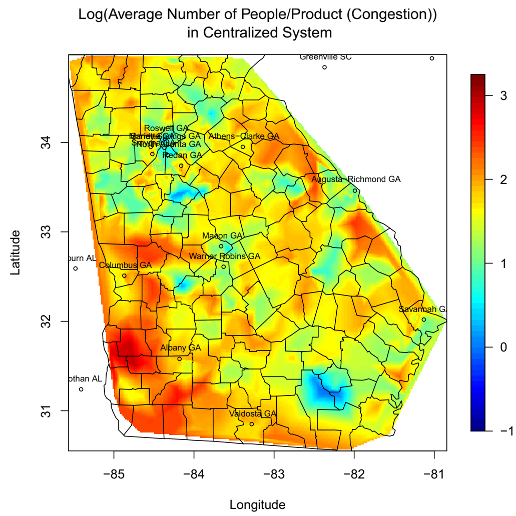


(b) Decentralized Distance

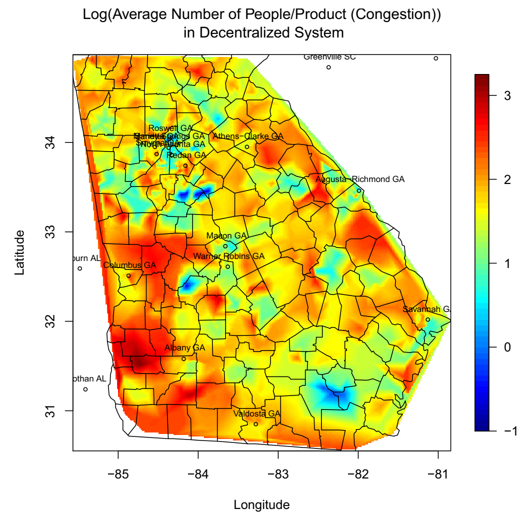


(c) Decentralized - Centralized Distance

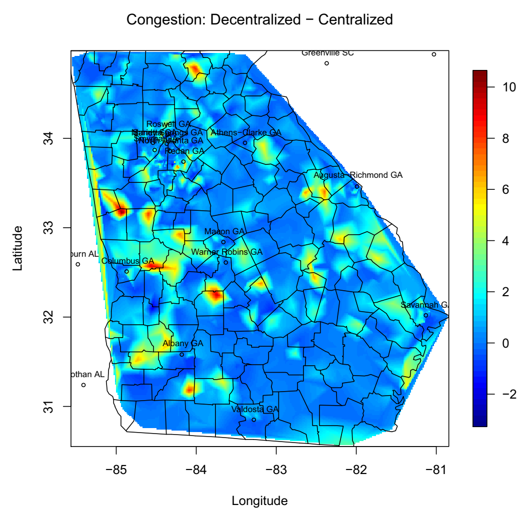
Figure 7: Comparison of distance between the centralized (a) and decentralized (b) models in Georgia, and difference between them (c).



(a) Centralized Congestion



(b) Decentralized Congestion



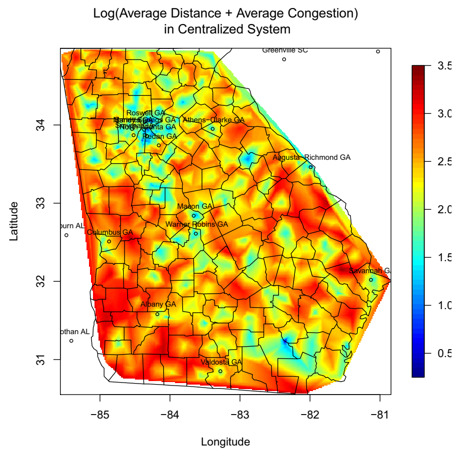
(c) Decentralized - Centralized Congestion

Figure 8: Comparison of congestion between the centralized (a) and decentralized (b) models in Georgia, and difference between them (c).

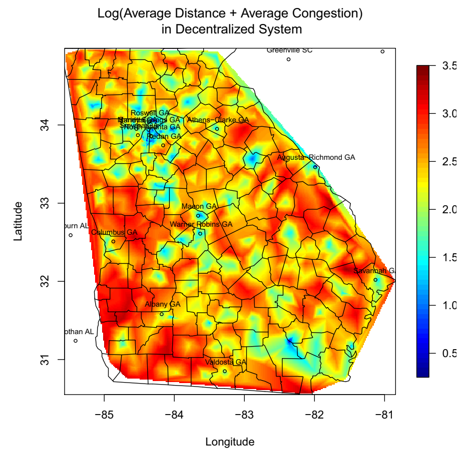
accounted for 1.9 percent of the total available product in the state. The under-utilized facilities include some with very small shipment quantities in areas where people had many facility alternatives, as well as some facilities that received an unusually large number of products that could not all be allocated given the number of individuals willing to travel to those facilities. In our computational experiments, we observe that the number of unallocated products increases as the community size in the decentralized model increases. This arises primarily as a result of communities avoiding facilities with very small product quantities, such as those that received the minimum shipment of only 100 products, because the community size overwhelms the product availability. Although increasing community size is a way to yield faster solutions or to solve models for larger regions, care must be taken to understand the implications of this modeling choice.

In Figure 9, we plot the sum of the distance and congestion experienced at the census tract level for both the centralized and decentralized systems. The range of centralized values is from 1.6 to 30.3, while the corresponding decentralized values are between 1.5 and 30.8. The differences between the systems using the aggregate measure are smaller than differences between either the distance or congestion measures alone; these differences are illustrated in Figure 9(c). From these maps, the ability of a centralized planner to smooth the effects of distance and congestion using a system-wide perspective is apparent, but the centralized approach ignores individual choices. The spatial variation in the decentralized results is more pronounced. In both systems, however, there is evidence of spatial differences in access to the product.

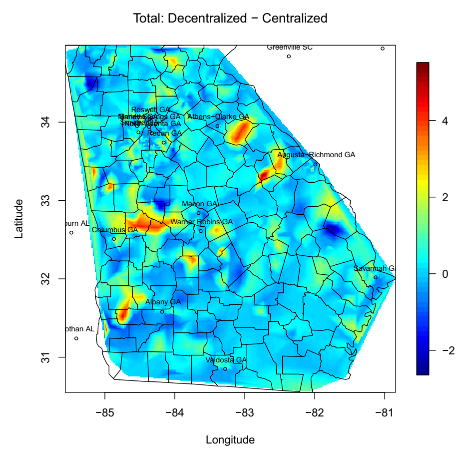
In addition to examining the differences between the centralized and decentralized systems, we consider the sensitivity of the decentralized results to the individuals' objective functions. Since actual data regarding individuals' choices about where to



(a) Centralized Distance + Congestion



(b) Decentralized Distance + Congestion



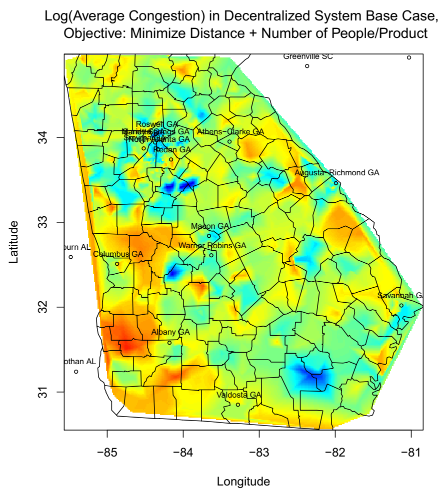
(c) Decentralized (Distance + Congestion) - Centralized (Distance + Congestion)

Figure 9: Comparison of distance plus congestion between the centralized (a) and decentralized (b) models in Georgia, and difference between them (c).

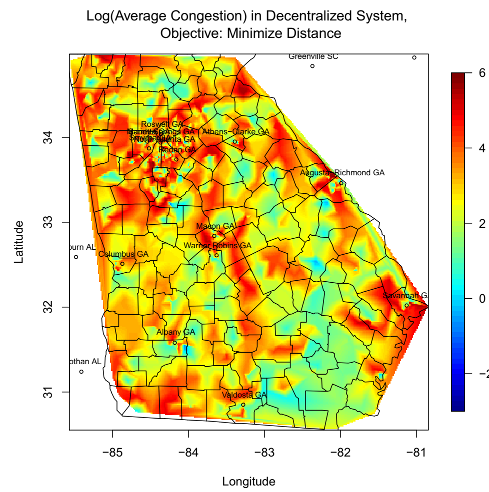
receive the product are not available, we investigate several possible decentralized objectives including distance only, number of people only, number of people per product, and weighted combinations of these factors. While the resulting measures of accessibility, namely distance and congestion, vary depending on the decentralized objective function, spatial variability in these measures persists. For example, the resulting number of people per product (or congestion) under several decentralized objective functions is illustrated in Figures 10(a)-10(d). The results under the original Equilibrium Condition given by Inequality (44) are shown in Figure 10(a) and re-scaled for comparison to the other objectives. Figure 10(b) illustrates the average congestion at the census tract level when the decentralized objective includes only distance. This value exhibits a great deal of spatial variability. (Note here that the scale also covers a wider range than in the other figures.) The patterns change as the decentralized objective moves from this extreme to an intermediate objective considering both distance and a weighted measure of persons per product (Figure 10(c)), and finally to the opposite extreme in which only the number of people at a facility is considered (Figure 10(d)). These results illustrate that spatial variability persists across a wide range of decentralized decision parameters. In what follows, we focus on the results for the base case that was introduced in Section 3.5.2.

3.7.2 Statistical Analysis of Socioeconomic Factors

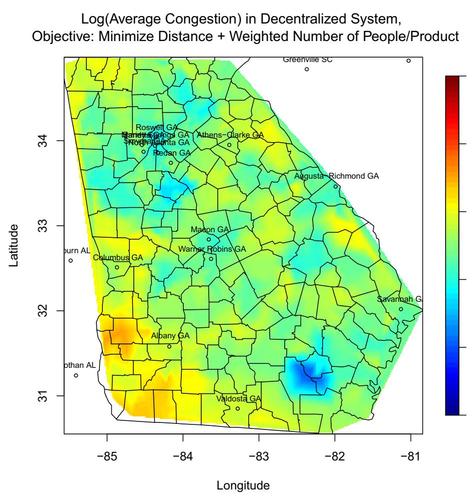
We use the space-varying coefficient model described in Section 3.5.3 to examine relationships between the congestion accessibility metric produced by the optimization models and a series of socioeconomic factors. In our analysis, we consider three different factors that reflect income: per capita income, median household income, and percent of population living below the federal poverty level. We examine six factors reflecting race and ethnicity: percent of population that is Black, percent of population that is Hispanic, percent of population that is Black or Hispanic, Black



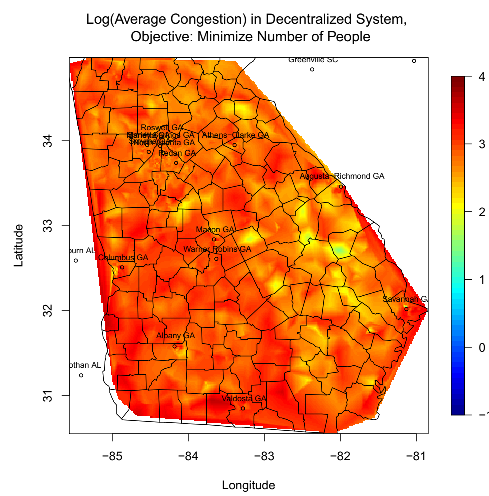
(a) Objective: Distance + Persons/Product.



(b) Objective: Distance Only.



(c) Objective: Distance + 5*Persons/Product.



(d) Objective: Number of Persons Only.

Figure 10: Congestion under various decentralized objective functions.

population counts, Hispanic population counts, and Black plus Hispanic population counts. Finally, we include population density and the number of potential service locations relative to population density in the models. For clarity of exposition, in the remaining discussion we refer to the latter factor as service availability. We use the natural log of all factor values so that differences can be seen more readily. Factors that are highly collinear (for example, per capita income and percent of population below the federal poverty level) are not used together within a model. The socioeconomic predictors are all measured at the census tract level. These factors are selected because income, minority group status, urban or rural location, and general availability of public services have all been associated with differing health outcomes or accessibility to other public services.

Here we present the results of the spatial statistical analysis examining the association of the aforementioned factors with the congestion accessibility metric. We implement the space-varying coefficient model with an intercept and four factors, examining the impact of different combinations of factors. We limit the models we consider to four explanatory factors for two reasons: using too many factors can cloud meaningful relationships and solving varying coefficient models is computationally challenging for a region of this size even with four factors. The analysis is done using R statistical software [91]. The model output includes the fitted space-varying coefficients for each factor, a shape (constant, linear, or nonlinear) for each factor, and model residuals. For factors that the model suggests are constant across space, we test the hypothesis that the factor is significantly different from 0 and report the p-value of that test. If the model suggests that factors are linear or nonlinear, the shape itself indicates that there is spatial variability captured by this factor and suggests significance.

Models with different factors are compared on the basis of several criteria. First, factor significance (including shape, as described above) and coefficient range are considered. Coefficients with greater absolute value indicate a greater contribution by that factor to the model. Second, we test for auto-correlation in the model residuals. If auto-correlation is low, this indicates that the factors in the model explain a substantial portion of the spatial variability in access to the product. Based on this comparison of candidate models, we select the one that most completely explains the spatial variations in the congestion accessibility metric.

Centralized Model Results

We find strong association between the spatial variability in the centralized system congestion and service availability, per capita income, percent of the population that is Black or Hispanic, and population density. As shown in Figure 12, the analysis indicates that the coefficient for service availability is constant across the state and its estimated value is -0.6975 (p-value = 0.000372). This means that as the number of potential service locations increases in relation to the underlying population, the congestion experienced decreases.

Per capita income is shown to have a nonlinear coefficient, meaning that the level of impact is different across the state. The range of coefficient values for this factor is [-1.2196, 1.2389]. Figure 11(b) maps the per capita income at the census tract level across the state; the corresponding factor coefficient fitted by the model is shown in Figure 13. Areas where the coefficient is smallest, which are blue on the map, experienced lower congestion than would be expected based on their own per capita income and the average effect that per capita income has across the state. On the other hand, dark red areas where the coefficient is largest experienced greater congestion than expected. The coefficients corresponding to per capita income exhibit both the largest magnitude and the largest range among the four factors in the model.

The percent of the census tract population that is Black or Hispanic also has a nonlinear effect on the congestion across the state, with a range of [-0.6808, 0.5168].

The factor and coefficient values are illustrated in Figures 11(c) and 14. While the magnitude of this impact is smaller than that of income or population density in many areas, the factor is still significant.

The effect of population density is nonlinear across the state, with coefficient values in the range [-1.034, -0.0922]. Its value everywhere, however, is negative. This means that congestion decreases as population density increases, although to varying degrees depending on the part of the state. The impact is greater in areas that are blue or green in Figure 15.

Figure 16(a) maps the residuals of the space-varying coefficient model for the centralized system congestion. These values represent the spatial variability left unexplained by the model. The blue colors in some locations indicate that the congestion is lower than our statistical model predicts, while patches of red indicate that the model under-predicts congestion. Overall, the statistical model does well; there is little evidence of auto-correlation in the residuals and the maximum correlation coefficient is less than 0.08.

Decentralized Model Results

The socioeconomic factors that best explain the spatial variability in system congestion in the decentralized system are service availability, percent of population below the federal poverty level, percent of the population that is Black or Hispanic, and population density. The factors are the same as those in the centralized model, with the exception that per capita income is replaced by percent of population below the poverty level.

The analysis indicates that the service availability has a linear shape across the state but that it is always inversely related to decentralized system congestion. The range of the coefficient is [-1.4441, -0.1279]. Figure 17 illustrates that the magnitude of this effect is greater as one moves southeast across the state. The average value of this coefficient, -0.786, is also smaller than the corresponding constant coefficient for service availability in the centralized model, indicating that the impact of this factor is slightly more pronounced in the decentralized system.

The percent of the census tract population living below the federal poverty level also exhibits a linear pattern, with coefficients ranging from -0.4809 to 1.1425. This indicates that in some areas, a higher fraction of people living in poverty is associated with decreased congestion, while in other areas the opposite is true. Lower coefficients are found in the north part of the state and generally increase as one moves south, as shown in Figure 18.

The third factor shown to be significantly related to decentralized system congestion is the percent of the census tract population that is Black or Hispanic. The coefficients of this factor are nonlinear across the state and their range is [-0.6531, 0.4376]. In the blue areas shown on the map in Figure 19, this factor is associated with decreased congestion, while those shown in red experience a positive correlation between the percent of the population in these minority groups and the decentralized system congestion.

Finally, population density is a significant factor. The corresponding coefficients are nonlinear, falling in the range [-2.6888, 0.9052]. Figure 20 illustrates some areas (in dark blue) of the state in which the population density is associated with a much lower congestion than would be expected based on the statewide average of the coefficient value, while in others (dark red) the opposite is true. In the decentralized model, the population density factor coefficients exhibit both the greatest magnitude and range among the four model factors.

The residuals of the best-fitting model for the decentralized system are shown in Figure 16(b). The map illustrates a number of locations where the model under-predicts (red) or over-predicts (blue) the congestion found by the optimization model. The residuals of this model exhibit some auto-correlation, with a maximum correlation coefficient of approximately 0.25. This implies that there are more unexplained inequities left in the decentralized model. This observation holds even when the same set of predictors is used for both scenarios.

3.8 Discussion

In both models, the results of this study indicate that there were geographical inequities in product allocation and accessibility across Georgia. Moreover, these differences were more pronounced in the model that explicitly captures individuals' choices than in the centralized model. Generally speaking, areas near the state's larger cities experienced lower values for both distance and congestion relative to the total product available, translating into better potential access. This is consistent with the mostly-negative correlation in the space-varying coefficient models between congestion and population density. Exceptions include those areas indicated by noticeably darker blues in Figures 7-9, evident especially in Figure 8. The central- and southwest parts of the state, as well as the central-east, appear to have experienced lower potential access. While some of this may have been mitigated by having access to product in neighboring states, preliminary results for the region-wide system indicate that this does not fully explain this observation.

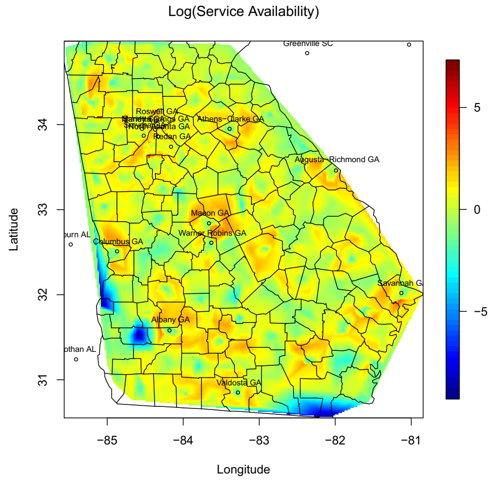
As has been shown when considering access to other public or health-related services, socioeconomic characteristics are associated with inequities in access to the product we study. The best-fitting statistical models for both the centralized and decentralized systems include factors representing the availability of candidate service locations, income, minority group population, and population density. Despite the fact that our service availability factor is based on all potential service locations, rather than on facilities actually distributing the product, this variable is significant in both models. Its negative correlation with congestion is indicative of the impact that differences in the underlying service infrastructure may have on the ability to mount an equitable distribution effort. Recruiting and retaining additional service locations in under-served regions is a long process, but this study points to the need for awareness of this challenge in planning for future response efforts.

There are several practical implications of this research. The results of the statistical analysis leave more unexplained inequities in the decentralized model. This reflects, at least in part, the local nature of decentralized decision making. Evidence of inequities persists even in the solution to a centralized optimization problem. The appropriateness of the decentralized or centralized frameworks in any particular context depends on whether individuals choose or a centralized authority can determine assignments of people to facilities. For example, states could adopt a distribution plan wherein individuals are assigned to service locations, much like the practice of assigning voters to polling places. Overall, this study demonstrates that both decentralized decision making and the state's choice of service locations and quantities have an impact on the resulting accessibility of the product at the census tract level. Inequities can arise even if the original allocation from the national level to each state is done pro rata, a seemingly equitable approach. To address this phenomenon in future distribution efforts, the national organization and state partners should consider the following actions:

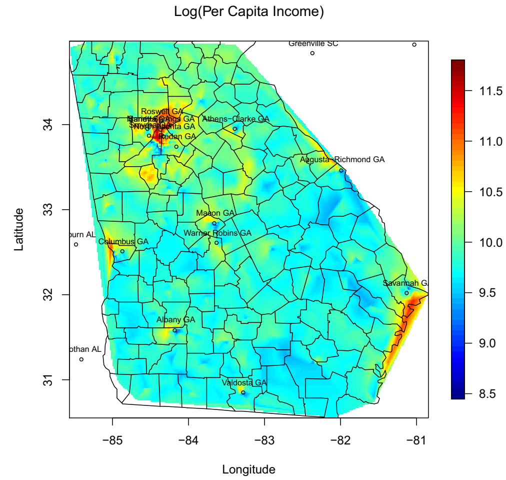
- Examine proposed distribution plans and product allocations to assess their implications for accessibility and equity prior to implementation.
- When recruiting facilities and community groups to serve as service locations during the planning or early response stages, make concerted efforts to involve those from areas shown to be inequitable in previous responses or other non-emergency initiatives.
- Consider using a model that identifies an equitable solution and adopts the equilibrium-obtaining congestion weights framework (from Chapter 2) to improve decentralized system performance.

3.9 Ongoing and Future Research

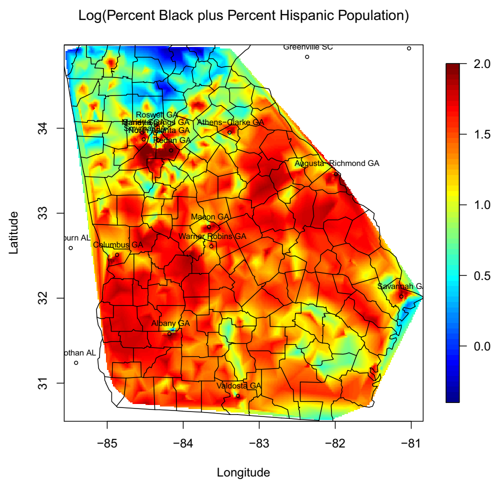
This research illustrates the importance of considering the overall system and accounting for individuals' choices during planning for large-scale distribution efforts. We are pursuing several additional areas of research related to this study. Currently, we are analyzing the output of the centralized and decentralized models presented here for a multi-state region in the southeast United States. This analysis demonstrates the applicability of our method to larger geographical areas and facilitates the comparison of inequities and important factors between states. We will investigate whether the factors that have been shown to be significantly correlated with outcomes in Georgia are similar to or different from those in other areas, as well as investigate patterns in the region as a whole. Following this analysis, we will proceed with other geographic regions. Although the current study focuses on the entire supply of the product during the shortage period, a subset of this product was intended specifically for children. Our future work will examine this specific subset in an effort to determine whether potential spatial access differed for children and how these differences are correlated with socioeconomic variables. Our aim in this series of studies is to identify regional similarities and differences, understand the factors driving these results, and develop recommendations that will lead to improved distribution efforts in the future.



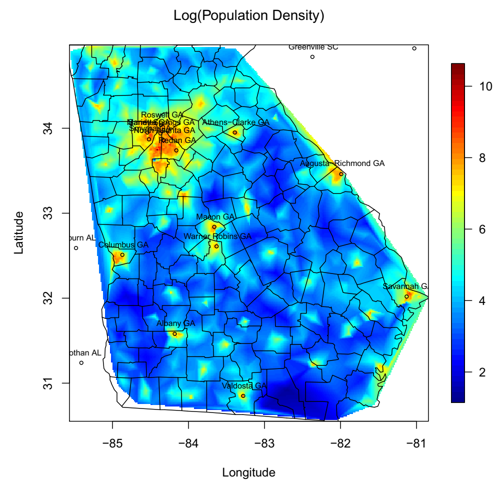
(a) Factor values for log(service availability).



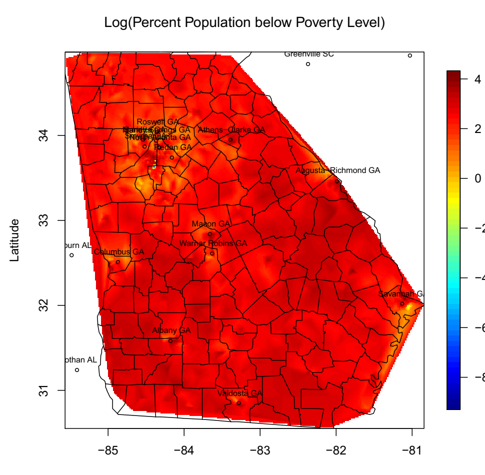
(b) Factor values for log(per capita income).



(c) Factor values for log(percent Black + percent Hispanic).



(d) Factor values for log(population density).



(e) Factor values for log(percent population below federal poverty level).

Figure 11: Explanatory factor values.

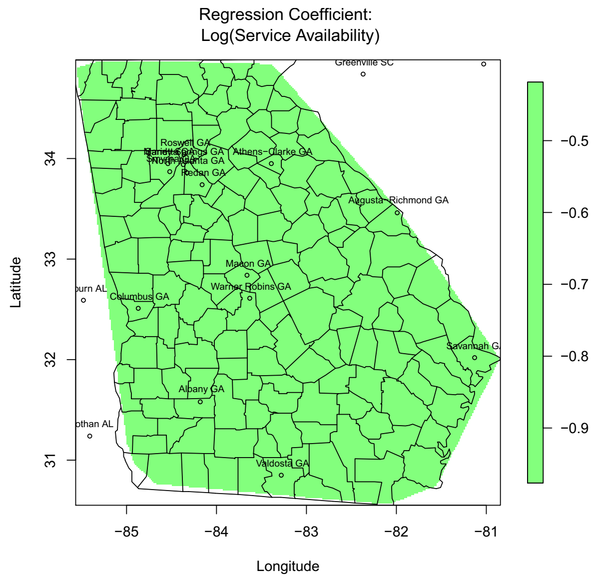


Figure 12: Coefficient values for log(service availability), controlling for income, percent population Black or Hispanic, and population density in the centralized system; coefficient shape is constant.

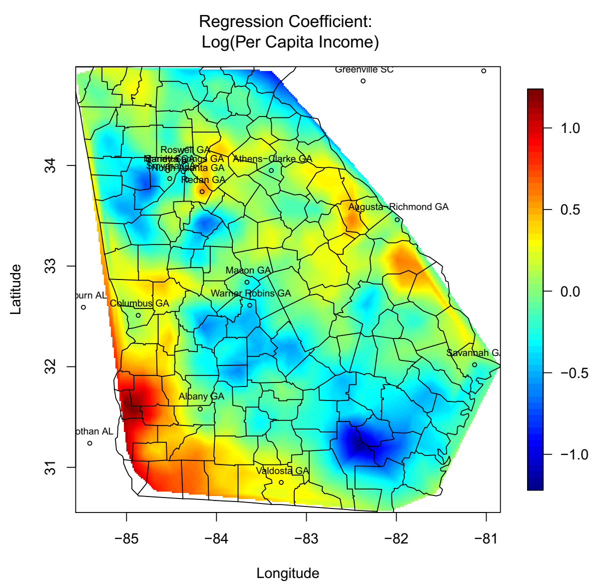


Figure 13: Coefficient values for log(per capita income), controlling for service availability, percent population Black or Hispanic, and population density in the centralized system; coefficient shape is nonlinear.

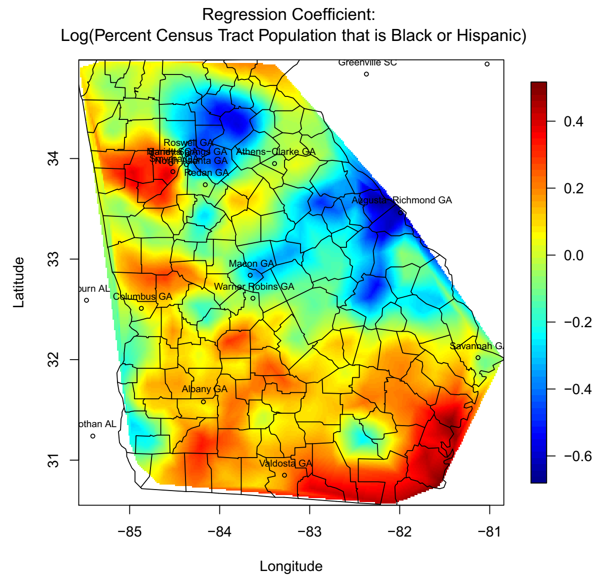


Figure 14: Coefficient values for log(percent Black + percent Hispanic), controlling for service availability, income, and population density in the centralized system; coefficient shape is nonlinear.

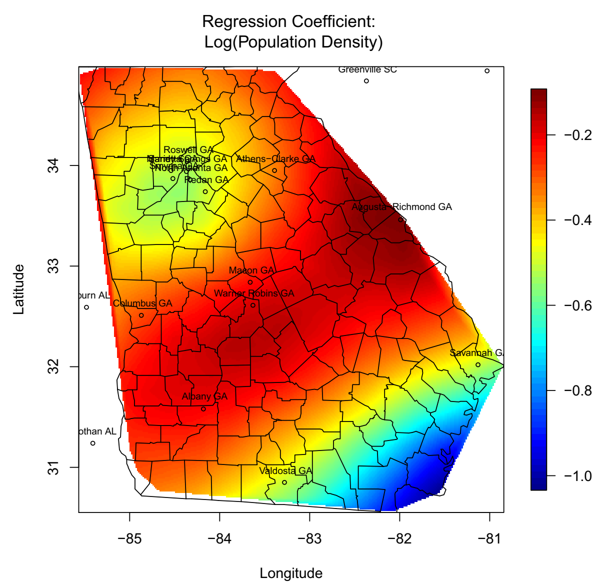
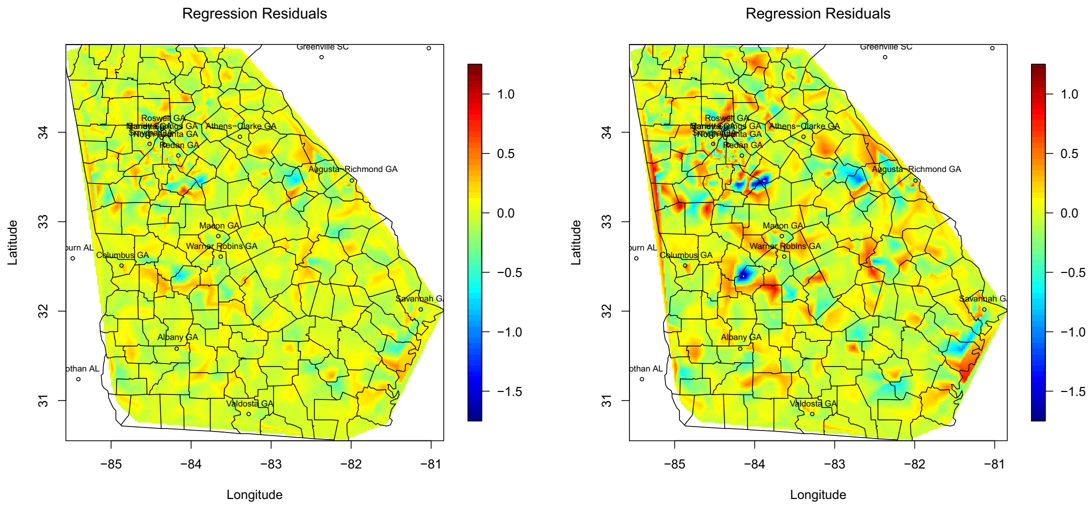


Figure 15: Coefficient values for log(population density), controlling for service availability, income, and percent population Black or Hispanic in the centralized system; coefficient shape is nonlinear.



(a) Centralized Residuals (b) Decentralized Residuals

Figure 16: Model residuals.

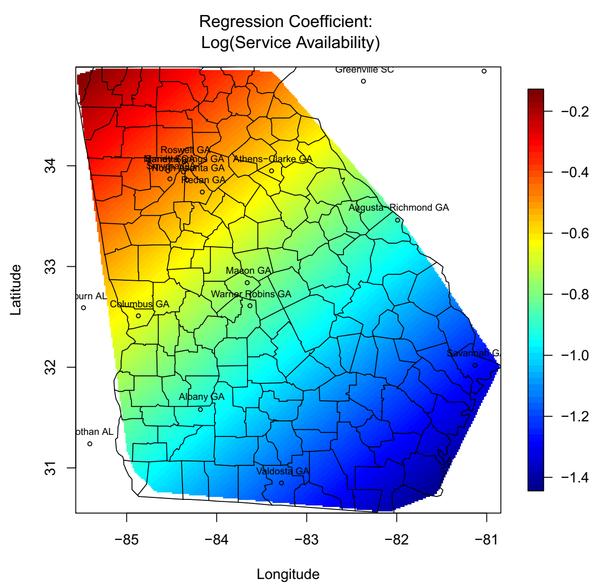


Figure 17: Coefficient values for log(service availability), controlling for percent population below poverty level, percent population Black or Hispanic, and population density in the decentralized system; coefficient shape is linear.

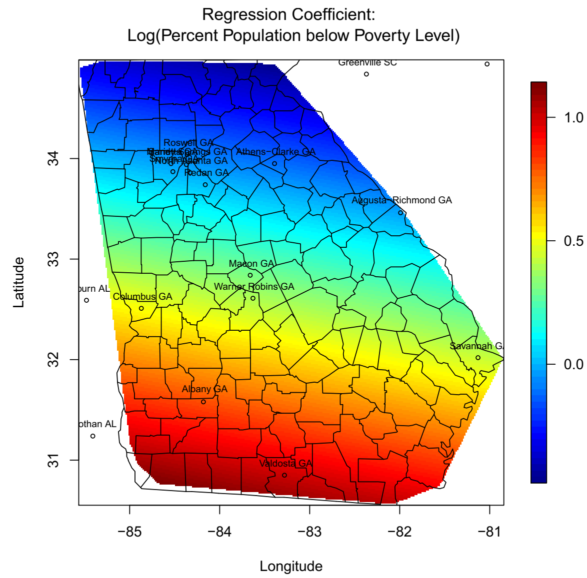


Figure 18: Coefficient values for log(percent population below federal poverty level), controlling for service availability, percent population Black or Hispanic, and population density in the decentralized system; coefficient shape is linear.

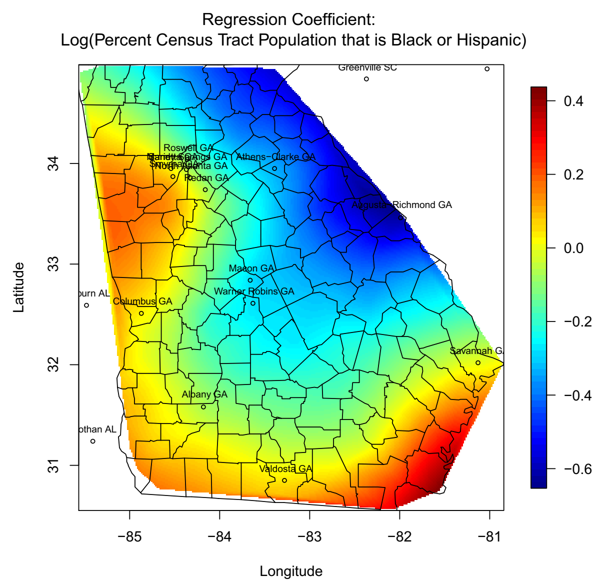


Figure 19: Coefficient values for log(percent Black + percent Hispanic), controlling for service availability, percent population below federal poverty level, and population density in the decentralized system; coefficient shape is nonlinear.

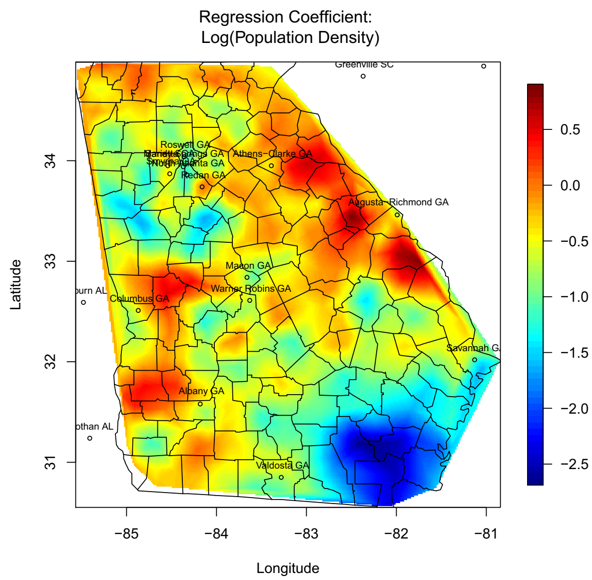


Figure 20: Coefficient values for log(population density), controlling for service availability, percent population below poverty level, and percent population Black or Hispanic in the decentralized system; coefficient shape is nonlinear.

CHAPTER IV

DISASTER RESPONSE LOGISTICS AT WAFFLE HOUSE RESTAURANTS

4.1 Introduction

Supply chain management practitioners and researchers have made significant contributions to improving the efficiency and effectiveness of a broad spectrum of organizations. Comparatively little work has been done to date, however, to apply logistics methodologies to the supply chains associated with disaster response. Arguably some of the most urgent challenges faced by our society include the development of effective methods to prevent, mitigate, and respond to emergency situations caused by natural or man-made disasters. Preparation for and response to disasters frequently require the coordination of resources and personnel from local, regional, national, and international governments, non-governmental organizations (NGOs), the private sector, and the affected population. As such, these challenges provide many opportunities for supply chain management professionals to make valuable contributions.

Some organizations have been successful in responding to disasters, and closer investigation of their practices helps move this field forward. Among those who are well-recognized for their ability to respond is Waffle House Restaurants, and this study aims to understand why this company has been successful. The chapter is organized as follows. The research objectives, case study methodology, and selection of Waffle House Restaurants as the emphasis of the study are described in Section 4.2; this section also provides a review of literature in this area. Section 4.3 describes the philosophy, structure, and processes associated with Waffle House Restaurants' hurricane response. An analysis of Waffle House Restaurants' success in terms of supply chain and crisis management tools, the types of decision tradeoffs inherent in disaster response, and lessons that translate to other organizations are detailed in Section 4.4. Concluding remarks are presented in Section 4.5. The material from this chapter has appeared in [29], and accompanying teaching materials are available [28].

4.2 Research Objectives and Methodology

This study is motivated by the impact of supply chain management methodologies in the day-to-day business operations in the private sector and the opportunity to improve their application in disaster response scenarios. It seeks to answer several questions:

- Are there private sector organizations that succeed in responding to natural disasters?
- Why is such an organization successful?
  - What type of crisis management and supply chain management techniques are used?
  - What are the roles of the different functional groups of the organization in the disaster response?
  - How do the functional groups collaborate within the organization?
  - How does the organization collaborate with other stakeholders?
- How can lessons from the experience of the organization be generalized to others?

A case study approach was selected to investigate these research questions. As described by Yin [107], the case study is a relevant research method when investigators seek to know how or why a phenomenon occurs, have little control over actual behavioral events, and are concerned with contemporary issues. Each of these characteristics describes the research topic at hand. Case studies also provide concrete examples of practical applications of management techniques, and thus, can help other organizations develop ways to translate theory into practice. By presenting this research as a case study, we pursue several important objectives. The study aims to highlight the processes and decisions that make Waffle House Restaurants a leader in hurricane response. Hence, while addressing the research questions posed, this study also stimulates improved disaster preparedness in other organizations by raising critical issues in disaster preparedness and response. Finally, the case study lends itself to the development of educational materials for use in academic and professional training environments. Developing greater awareness and competence among supply chain professionals is a critical step toward improving disaster response and the case study approach helps achieve that goal.

After framing the research questions and selecting the case study methodology, Waffle House Restaurants was selected as the focus of this study. The company began with a single store in 1955, but now has restaurants in 26 states and a dense presence in the Southeast region of the United States. This geographic presence makes the stores vulnerable to the effects of hurricanes and has led the company to develop nationally-recognized disaster response processes. Their proactive approach earned them recognition from ABC News following Hurricanes Katrina and Rita for their "rapid ability to provide relief in disaster," alongside Wal-Mart, Home Depot, and Lowe's [44]. In contrast to these other companies, Waffle House Restaurants is privately owned. It does not have its own transportation fleet and has comparatively fewer resources at its disposal, making internal and external collaboration a key component of its success in hurricane response. Unlike other companies that participate in humanitarian response, such as major global logistics firms United Parcel Service (UPS) and TNT, logistics is not the primary business of Waffle House Restaurants.

The company's use of supply chain management tools in support of its response efforts thus provides insights for others who strive to do the same.

4.2.1 Supply Chain Management and Disaster Response in the Literature

Standard supply chain management problems such as resource allocation, transportation planning, and inventory management all develop a new level of complexity in the context of disaster response. Infrastructure and resource availability is uncertain, and demand can vary widely both in volume and location. Decisions must often be made very urgently and with limited information. In this context, there is a need for complementary tools from both crisis management and supply chain management. Before describing the ways that Waffle House Restaurants has dealt with some of these challenges, we summarize previous work in these areas.

The classical text on crisis management is Weick and Sutcliffe's *Managing the Unexpected* [103], in which the authors examine a number of what they call high reliability organizations, such as nuclear power plants, firefighting crews, and aircraft carriers. These high reliability organizations face substantial risk of disastrous events but routinely perform very well. From these successful organizations, five practices are derived that are essential for others that seek to perform as well and manage crises effectively. These practices include a preoccupation with failure, in which error reporting is encouraged and even small problems addressed; a reluctance to simplify interpretations so that all perspectives are considered; a sensitivity to operations, with a focus on detailed workings of the organization as opposed only to strategy; a commitment to resilience by building individuals' capacities and returning to a state of preparedness as soon as possible; and a deference to expertise, empowering those who have experience in a needed area regardless of their rank.

Pearson et al [76] describe five phases of effective crisis management: crisis prevention, preparedness, containment, recovery, and learning. Later, they emphasize the importance of a systems approach to crisis management: "If a company could adapt already existing systemic approaches to include its crisis-management activities, then it might be more likely to address the crises proactively rather than after the fact." In addition to discussions of preparedness approaches that make use of specific plans, other researchers have examined characteristics that make organizations inherently resilient and seek to measure this resilience potential [64, 87]. Somers [87] demonstrates that resilience potential correlates with certain activities that can be influenced by managers, such as organizational structure, participation in community planning activities, and the extent to which management seeks information about risks.

Given our focus on supply chain management tools within the context of disaster response, the literature on supply chain disruption and risk management also provides insights. Tang [89] provides a detailed survey of work in this area. He notes that the majority of the existing models deal with operational risk rather than disruption risk, and while many companies recognize the importance of risk assessment, few invest time and resources to mitigate disruption risks. The author highlights several important open research areas to develop quantitative models that can help companies better manage disruption risk. Of particular relevance to this case study are the consideration of different objective functions and management targets (besides expected cost), as well as strategies for demand, supply, and product management.

Tomlin [93] quantifies the value of mitigation and contingency strategies in the face of supply-side disruption risks. The author derives optimal policies for a single-product, two-supplier case in which one supplier is fully reliable and the other is unreliable but less expensive. Inventory mitigation, supplier choice mitigation, and contingent rerouting (by choosing another supplier) are examined. These theoretical results provide insights for the kinds of policies that may be effective in practice. Despite much work in the area of risk and uncertainty, research in the area of supply chain disruption as a result of disasters remains an important and growing field [57].

Several articles describe supply chain operations in the context of humanitarian response. A detailed overview of the challenges of humanitarian logistics, in particular the interaction of many stakeholders, is provided by [97]. This study and a related article [92] also highlight the opportunity for humanitarian agencies and private companies to learn from one another to improve operations, emphasizing the successful partnership between the World Food Programme and TNT. Others [12, 50] have addressed the important role that information management and dissemination play in response efforts, while [9, 10] examine questions of inventory management in disaster response supply chains. [60] describes improvements made in the FEMA procurement system since Hurricane Katrina, including the use of both pre-positioning and pre-negotiated contracts with suppliers of relief necessities. Despite the research that has been done to date, [59] emphasizes the need for additional quantitative research to support planning and processes for disaster response.

4.2.2 Waffle House Restaurants Hurricane Response

This case study differs from much of the previous literature in that it describes the efforts of a particular private company, as opposed to a humanitarian agency, in preparing for and responding to natural disasters. The study showcases the Waffle House Restaurants hurricane response philosophy, the processes the company follows in preparing for and responding to hurricanes, and some of the lessons learned throughout the years of response activities. The case study details the responsibilities of the different functional groups of the company. It also describes ways that Waffle House Restaurants have improved hurricane response processes as a result of lessons learned from previous hurricanes.

This case study contributes to the literature in crisis management and humanitarian logistics by providing concrete examples of effective techniques in practice.

We relate the outcomes of our original research questions regarding the company's success to the underlying theoretical framework in these disciplines. We demonstrate that Waffle House Restaurants exhibits several of the qualities of high reliability organizations and that the company has focused much effort on developing systematic processes for the phases of effective crisis management. This case study illustrates that planning for supply chain disruptions and using logistics methodologies, such as supply and demand management and inventory planning, can contribute to effective response in humanitarian crises. Finally, it provides lessons for companies in the private sector as well as government and humanitarian agencies and raises questions for future research in this area.

4.3 Case Description

Tom Forkner and Joe Rogers, Sr., opened the first Waffle House Restaurants store with a commitment to outstanding service, both to their customers and to their employees. This dedication to the community is still witnessed today, and perhaps is most evident in the company's philosophy toward hurricane response. In this case study, we detail the company philosophy and describe their preparation for hurricane season, their response process, and the lessons they have learned to improve the process through the years.

4.3.1 Company Background

In 1955, the first Waffle House Restaurants store was opened in Avondale Estates, outside of Atlanta, GA.[1] More than 50 years later, it has grown to approximately 1600 stores in 26 states. 650 of these are owned and operated by franchisees, with the remainder managed by the company itself. Waffle House Restaurants has various

[1] Company background was obtained through personal interviews with Waffle House Restaurants personnel and by consulting the Waffle House Restaurants media kit, available upon request at http://www.wafflehouse.com/faq.asp.

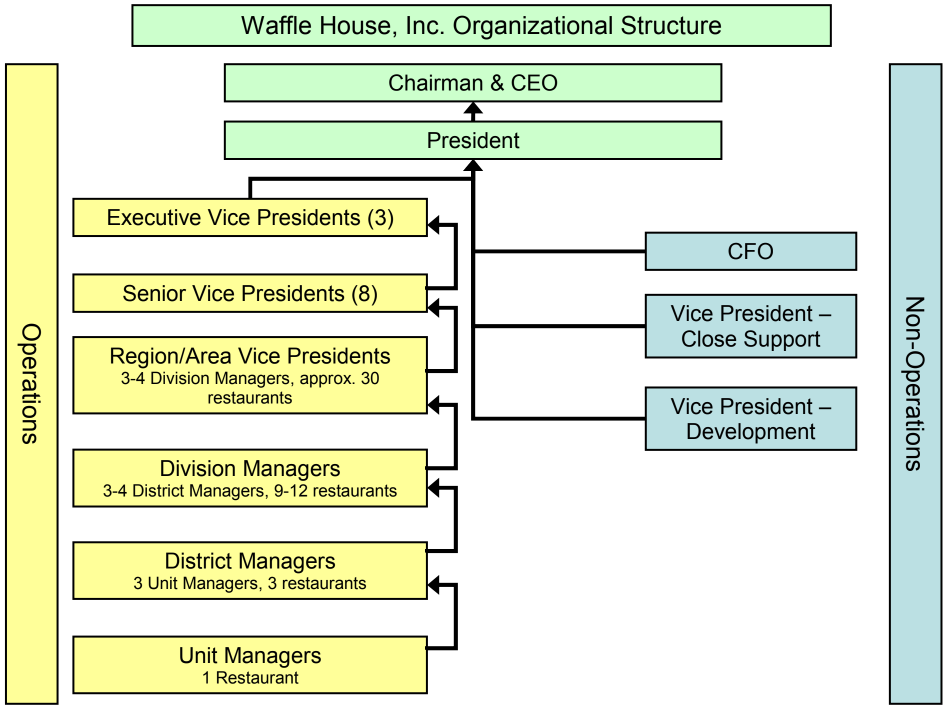


Figure 21: Waffle House, Inc., Organizational Structure.

offerings, including breakfast items, T-bone steaks, hamburgers, country ham, pork chops, and grits.

Today, employees in stores across the country contribute to serving Good Food Fast® in Waffle House Restaurants open 24 hours a day, 365 days a year. The company is headquartered in Norcross, GA, and the Southeast remains a large market. Therefore, a significant percentage of Waffle House Restaurants are subject to the effects of hurricanes. The company's hurricane preparations involve employees from all of the functional groups in the organization.

The specific roles of each group in the hurricane response process are detailed in Section 4.3.4. To assist in understanding the relationship between the groups, Figure 21 illustrates the organizational structure of Waffle House Restaurants. There are two major divisions of the company: operations and non-operations. The operations division runs the company- and subsidiary-owned restaurants. Three unit managers, each responsible for one restaurant, report to each district manager, and three or four district managers report to each division manager. The region/area vice presidents are each responsible for three or four divisions, for a total of approximately 30 restaurants. The region/area vice presidents report to eight senior vice presidents, who in turn are responsible to three executive vice presidents. The final roles in the managerial structure are those of the president and of the chairman and chief executive officer, who have responsibilities for both operations and non-operations.

In the non-operations division, there are three primary areas. The chief financial officer, the vice president of close support, and the vice president of development each report to the president. The finance team is responsible for accounting, finance, franchise development and support, tax, and stock tasks. Close support manages marketing and communications, human resources and training, and operations control, which supports company operators. The development team is responsible for real estate, construction, and equipment and property management.

4.3.2 Disaster Response Philosophy

The processes used by Waffle House Restaurants to respond to hurricanes have evolved over the course of many years, but one fundamental theme directs all the response activities. "Nothing good can come from a closed Waffle House after a hurricane - not for us, not for the community, not for the associates." This statement from former Waffle House Restaurants president and chief operations officer, Bert Thornton, summarizes the company's disaster response philosophy. The company is committed to getting back into the affected areas, opening stores as quickly as it is feasible and safe to do so, and helping the local economy and restaurant associates rebuild. Despite the fact that operating in these conditions presents significant economic and logistical challenges, Mr. Thornton explains, "Our position is this: those customers and those associates are there for us in the good times, so it's our responsibility to be there when times are tough. We do not take a back seat; we don't subscribe to the theory that you just wait until everything is easy to do and then open up the doors. We're always the first ones in."

Because of this commitment, the company's reputation now precedes their arrival into an affected area. The National Guard and local authorities welcome the arrival of Waffle House Restaurants personnel, recognizing that they provide an essential service in the form of hot meals for National Guard personnel, local emergency responders, electricians, and community members alike in the aftermath of a storm. Waffle House Restaurants' commitment and philosophy have informed the development of an integrated hurricane response process. The effectiveness of the response stems from proactive preparation and continuous improvement.

4.3.3 Hurricane Response Cycle

Preparation for hurricane response, for which the company dedicates significant time and resources, is part of Waffle House Restaurants' larger disaster planning. Preparation begins in the spring, prior to the beginning of the hurricane season. A major weather event triggers the response systems, and lessons learned are documented for future seasons.

Many years ago, hurricane efforts were a response *task*, but Waffle House Restaurants senior managers now consider them a response *event*. The change is reflected in the number of people and resources that are dedicated to hurricane planning. The leadership team is comprised of senior management from each of the company's functional areas, and the team participates both in the annual preparation meeting and on-the-ground response. Many are 20- or 30-year veterans of the Waffle House Restaurants business. Though the response processes are well-documented, director of purchasing, Greg Rollings, explains, "They won't be lost if they lose the checklist."

4.3.3.1 Pre-season Preparation

The Atlantic basin hurricane season officially extends from June 1 through November 30 [69]. To make a quick and effective response possible, the Waffle House Restaurants hurricane response cycle begins with pre-season preparation. At the annual hurricane preparation meeting each May, all members of the hurricane response team gather at the corporate headquarters. Lessons learned from the prior season are reiterated, key responsibilities are confirmed, contact information and procedures are updated, and the response protocol and timeline are reviewed. In addition to the annual meeting at headquarters, communication and training occur at all levels of the organization. Associates at all restaurants receive information about how to prepare their homes and families for an imminent hurricane, as well as how to contact the company to confirm their safety and obtain information about returning to work.

Prior to the onset of hurricane season, response processes are also reviewed with Waffle House Restaurants' key vendors with whom the company interacts on a regular basis. Arrangements are also made to obtain supplies specific to hurricane response, including fuel and portable toilets. These items are critical because electricity and water are often unavailable following a storm. At this time, other key assets including generators, vehicles, and communications equipment are secured either through purchase or lease agreements.

When Waffle House Restaurants reopen following a hurricane, they serve a limited menu rather than the full normal selection. The items and prices for this hurricane menu are determined prior to the hurricane season. These are filed with the appropriate government authorities in each hurricane-prone state to document the restaurant's commitment to consumer protection practices. Prices are fixed to the current pre-season value and the lowest applicable tax rate is used. The resulting values are then rounded down to the nearest nickel to make it easier for associates to conduct transactions if cash registers are not operational. The hurricane menu is used to simplify operations in the immediate aftermath of a storm and is discussed in more detail later in Section 4.3.4.4.

4.3.3.2 Impending Storm Preparation

During hurricane season, weather in the region is monitored daily. If a landfall is deemed likely at Category 3 strength or above [70], the hurricane response process is initiated. The first step in the process is the designation of a commander-in-control. This person, stationed at the corporate headquarters, is responsible for coordinating all of the response activities.

When a storm is imminent, the purchasing group executes a number of actions to prepare for store closure and reopening. In particular, the timing, destination, and quantity of food deliveries must be managed with suppliers. In addition to coordinating the food supplies in the days leading up to landfall, the purchasing department also secures recreational vehicles (RVs) and a refrigerator truck, as deemed necessary. The RVs are used to transport and house Waffle House Restaurants personnel who will enter the affected area immediately after the storm passes, while the truck is used to transport supplies from stores that cannot or will not reopen immediately to those that will. These vehicles are dispatched to staging locations closer to the storm area in the day before landfall. Along with the dispatch of the RVs and the refrigerator truck, the response teams from each of the functional areas are mobilized in the days leading up to the storm.

4.3.3.3 Post-storm Action

Smaller hurricanes and tropical storms are handled on a case-by-case basis and may be addressed by a smaller team or through the company's standard crisis procedures. However, for storms that are large Category 2 hurricanes or above and thus affect a significant number of restaurants, the Waffle House Restaurants hurricane team aims to have personnel in the affected area 12 hours after the storm has passed.

These responders travel from their staging locations and immediately begin assessing damage and implementing a recovery plan. Each functional group within the company has specific responsibilities during the recovery, which are detailed in Section 4.3.4. The recovery plan draws on the documented hurricane response procedures.

4.3.3.4 Return to Normalcy

Waffle House Restaurants is committed to speeding the return to normal operations for the sake of its employees and the communities in which it operates. Depending on the scope of the hurricane impact, this process may take less than a week or more than a year. The company is able to reopen stores more quickly than many other businesses because of their proactive approach developed over time. Lessons learned in each storm are used to improve future preparedness. Many of the processes outlined in the remainder of this case study are the result of experience in responding to hurricanes throughout the years. The company responds more quickly, has more resources dedicated to the response, and has identified what supplies and tasks are critical to a successful recovery operation.

4.3.4 Functional Area Responsibilities

In this section we describe the responsibilities of each functional group in the days immediately preceding and following a hurricane that affects company-managed stores. The primary areas of responsibility include the Purchasing, Construction and Equipment, Operations, People and Information Technology, and Control functions. The response efforts are coordinated by a commander-in-control stationed at corporate headquarters in Norcross, GA. Senior managers are involved in every step of the response process and are among those that are on the ground immediately following a storm. In the case of franchisees, corporate leaders make available any resources requested, but the franchisees lead the response process.

4.3.4.1 Commander-in-Control

Stationed at corporate headquarters, the commander-in-control communicates across all functional groups and mobilizes corporate resources to meet the needs of the personnel in the affected areas. This is a very intense responsibility, often requiring 18-hour work days. The company has identified a number of people with experience in filling this position, and the person initially appointed to this role typically passes the responsibility to another after the first several days of the recovery effort.

4.3.4.2 Purchasing

The purchasing group has developed a timeline to assist with the many tasks necessary to prepare for and respond to a hurricane. Five days before the anticipated landfall, communications with suppliers are critical to ensure that the suppliers have adequate inventory of the emergency items (which are not part of typical orders) Waffle House Restaurants will need. Items such as to-go supplies, paper towels, cleaning materials, hand sanitizer, ice, bottled water, and canned drinks are all needed in much greater quantities following a hurricane than under normal circumstances. Communicating with suppliers is also critical so that they can begin planning for drivers and deliveries of supplies immediately after the hurricane.

As landfall nears and the areas likely to be affected are more clearly identified, suppliers are notified to suspend delivery to areas that are being evacuated. The company's primary supplier has ready access to hurricane-affected areas because it also provides food for hospitals. However, other suppliers are often unable to mobilize quickly following a storm. To remedy this, purchasing personnel alert other suppliers in advance of an oncoming storm and stage extra product in restaurants that are outside the storm path but near the affected markets. This enables Waffle House Restaurants personnel to draw from these stocks to supply affected stores, transporting the goods themselves until normal shipments from these suppliers can resume. For example, two days prior to expected landfall, the company's bread supplier makes its last deliveries to the affected area. In addition to the standard delivery, the bread supplier delivers safety stock to stores that are out of the storm's path but nearby. This bread supplies the reopened stores in the days immediately following the hurricane.

When a storm is imminent, purchasing personnel secure recreational vehicles (RVs) to provide sleeping quarters for the teams arriving on the scene first. These teams are staged in locations that will ensure their safety while still allowing quick access to affected areas. For example, during a Gulf Coast hurricane, response teams are staged to the east and west of the predicted storm path. This enables them to enter the affected area as the storm pushes north.

Two days prior to projected landfall, the purchasing group also places an order for the first truck containing "first-wave supplies": ice, bottled water, canned soft drinks, to-go supplies, and cleaning supplies. The contents of the first-wave truck have been standardized based on experience in past hurricanes. This truck is staged and ready to deliver supplies on the day after landfall as soon as the operations group has determined which units will open first.

The first restaurants that are opened are able to operate with the supplies from the first-wave truck, their remaining food, and items recovered from surrounding restaurants. As additional restaurants are identified for opening, the second wave of supplies is ordered. Until recently, when a store was identified for reopening immediately following a hurricane, each restaurant manager was responsible for determining the food and supplies that were necessary. In the chaos, this frequently led to a mismatch between orders and requirements. After Hurricane Ivan in 2004, purchasing personnel developed a hurricane inventory sheet with a par level for each item. This tool enables managers to assess their current inventory levels and order up to the par level for each item. Standardizing this process has improved Waffle House

Restaurants' ability to effectively open and operate stores quickly after a storm.

In addition to managing food and restaurant supplies, the purchasing group secures a refrigerator truck and RVs for use in the response effort. The truck is used to transport food from nearby restaurants to those that are selected to reopen first. The RVs provide mobile control centers as well as housing for response personnel.

4.3.4.3 Construction and Equipment

The construction group has two primary responsibilities in the immediate aftermath of a hurricane: assess damage to all affected restaurants and manage refueling for all on-location responders. To execute these tasks, this group interfaces with several other functional areas before and after the storm. Prior to hurricane season, the construction and purchasing groups ensure that RVs are available to serve as sleeping quarters for response personnel and as command centers for the response. Following a storm, construction leaders work with operations managers to determine which stores can be reopened and in what order. These decisions are based on the physical condition of the restaurants; the availability of staff, food, and supplies; proximity to other restaurants; and proximity to major transportation routes. The equipment group is closely involved in these tasks, as well, arranging for generators and other equipment necessary to reopen restaurants.

Supplies of fuel are scarce and demand is high following a major hurricane. Frequently, Waffle House Restaurants must rely on generators to operate restaurants until electricity is restored. In addition, the response teams must travel between all the restaurants in the affected area so fuel for vehicles is a necessity. The company has developed processes to obtain fuel by establishing relationships with suppliers in the pre-season and making arrangements when storms are imminent. This is a joint responsibility of the construction and purchasing teams.

4.3.4.4 Operations

On a day-to-day basis, the operations group is responsible for running the restaurants. Following a hurricane, operations personnel work with the construction group to identify which restaurants to reopen and in what order. They are also responsible for the on-site tasks that are necessary for operating the stores, such as food preparation, customer service, and cleaning.

Until Hurricane Hugo in 1989, Waffle House Restaurants that were reopened in the aftermath of a hurricane offered the same menu as they did under normal operations. But this poses a challenge in post-hurricane conditions, especially for made-to-order breakfast items. Waffle House Restaurants president and chief operations officer, Bert Thornton, explains the advent of the hurricane menu:

> I was standing in the middle of it and listening to people call out orders like, "Let me have an order over-medium plate scattered, smothered, covered, chunked, diced, topped, peppered, and capped," and our people were going nuts. So I actually got up on a chair and I said, "Ladies and gentlemen, I've got good news and bad news for you. The good news is everybody's gonna get fed. The bad news is -," because I had hand-written on several pieces of paper for each salesperson - I said, "Your salesperson will show you what you can order. . . . it's a limited menu. You can have anything you want; now here's the list of things that you're gonna want."

Since then, the hurricane menu has changed as Waffle House Restaurants personnel observed what worked well and what did not. For example, bacon is not available in a post-hurricane situation because it takes up a lot of valuable grill space and requires long cooking times. However, ham was added because it is fast. The decision about what to offer on the hurricane menu requires a consideration of the space and time available for cooking, both of which are at a premium in the aftermath of a storm. Waffle House Restaurants are typically the first, and often remain the only, restaurants open for some time after a major hurricane. Law enforcement, emergency responders, and local citizens are anxious for a hot meal. While they are grateful for any food, customers do have preferences and menu diversity is important. Choosing the menu items becomes a balance between resource availability and customer utility. The hurricane menu does not differ significantly from the normal menu, but it is a combination of what people want most and what is easiest to prepare.

4.3.4.5 People and Information Technology

The people group has a number of important tasks following a hurricane. First and foremost, all employees must be accounted for. Prior to hurricane season, all associates are provided with a key chain attachment that is printed with a phone number to call to confirm their own location and the status of their restaurant. Working with operations and construction personnel, the people group helps redeploy associates to the stores that will be reopened first. If an employee is available, but his or her home restaurant has not yet reopened, that associate often works in a nearby restaurant until the home restaurant comes back online. The people group also works with the control group to manage payroll processes in the days following a hurricane.

At the onset of a response, at least one person in the people group must also secure hotel rooms for the response personnel and displaced associates. This is a difficult task given the high demand for rooms. To help address this issue, Waffle House Restaurants purchased a recreational vehicle (RV) following the 2005 hurricane season. The RV is outfitted with satellite capability to support internet and phone communications. This helps alleviate another challenge that arises in large-scale hurricane response - communications among people in the affected area as well as with the corporate headquarters. Because cellular and landline phone service are frequently unavailable in the aftermath of a storm, satellite capabilities greatly simplify communications.

Information technology personnel are tasked with addressing hardware issues and securing any necessary replacement parts. They also assist with the shutdown and restart processes for restaurant computer systems. Finally, a corporate psychologist plays a vital role in response efforts and is available both to local associates and to responders.

4.3.4.6 Control

The control function has two primary challenges in the aftermath of a hurricane. The first is accounting for food that was already in restaurants prior to the storm. As part of the closing process, most food at each restaurant is stored in the freezer, where it stays frozen for a short period even if electricity is lost. If a store is chosen for immediate reopening, this food is still available for use. However, if the store cannot be reopened, the food may be transferred via refrigerator truck to another restaurant. The control group is tasked with tracking this movement of food to help maintain inventory accuracy for each store. The second challenge faced by the control group is administering payroll. Waffle House Restaurants pays hourly associates every Sunday and wages are paid in cash. Following a hurricane, banks are often closed, so cash must be brought in from outside the affected area.

4.4 Case Analysis

Having examined the hurricane response processes used by Waffle House Restaurants, we now analyze these practices in light of the original research objectives. This analysis includes a description of the tools used by the company, a discussion of decision tradeoffs and collaborative efforts, and a summary of lessons to assist other organizations.

4.4.1 Tools for Success

The discussion of Waffle House Restaurants' hurricane response processes provides an affirmative answer to the first question posed by this study, which sought to determine whether there are private sector organizations that succeed in responding to natural disasters. The company's success is measured in terms of the time to reach the affected area, the time to return stores to operation, and the ability to put people back to work.

Reports from local and national news media attest to the results that the Waffle House Restaurants hurricane response process achieved, specifically in the wake of the devastating 2005 season. From Mississippi, where two restaurants sustained damage and seven others were completely destroyed by Hurricane Katrina: "Waffle House is rebuilding along decimated U.S. 90 with an efficiency apparently matched only by the casino industry" [37]. The Pascagoula, Mississippi, location was reopened less than three days after Hurricane Katrina raged through the area. A restaurant industry professional reporting for a trade magazine stated, "A veteran of restaurant openings, I did not expect the event to be so momentous. But that Waffle House, by opening its doors, assured the community that there was a chance life could return to normal. It provided comfort and a sense of home amid unfathomable devastation" [84]. Reports from Port Arthur, Texas, in the days after Hurricane Rita carried a similar theme: "Since the hurricane came ashore, very few people in Southeast Texas have been able to enjoy the restaurant experience, mainly because there's no restaurant open to experience ... The Waffle House ... was among the first restaurants to open in a storm-ravaged city that's thirsting for electricity" [53].

While these news articles illustrate the results, they do not highlight the crisis and supply chain management tools the company uses to achieve them. Examining the steps of the company's hurricane response cycle, however, reveals many of Weick and Sutcliffe's [103] practices for high reliability organizations in action. The continuous learning approach helps address even small problems in hurricane preparedness so that larger failures do not result. Waffle House Restaurants is certainly an organization that is sensitive to operations. Senior management team members started their Waffle House Restaurants careers in the kitchen and the hurricane plans reflect the detailed on-the-ground approach. The company measures its effectiveness by how quickly it is able to return to normal operations, demonstrating the commitment to resilience. As evidence of the practice of deferring to expertise, the people on-site have the authority to make decisions regarding the response effort with little need for information to travel up long chains of command that can hinder decision making in larger organizations.

We also see that each of the five phases of effective crisis management described in [76], namely crisis prevention, preparedness, containment, recovery, and learning, are represented in the Waffle House Restaurants' hurricane response cycle. While the extent of crisis prevention and containment are limited in the case of hurricanes, these do play a role as restaurant locations ready their buildings and personnel for anticipated storms. Much of the company's success stems from preparedness and learning, both of which enable more efficient recovery. Observations of Waffle House Restaurants hurricane response process provide empirical confirmation of Somers' [87] findings that such activities are correlated with resilience potential.

Supply chain management tools have been implemented as Waffle House Restaurants' personnel learned from hurricanes over time. Examples include pre-negotiated contracts and agreements with suppliers, inventory pre-positioning plans, procurement strategies, and inventory management tools such as par level policies. One challenge that the company faces is that it does not own its own transportation fleet. Waffle House Restaurants have addressed this by building strong relationships with suppliers and by renting refrigerator trucks to help transport supplies after hurricanes. Waffle House Restaurants must also take into account the perishable nature of its inventory in its hurricane supply plans and inventory management. These practices illustrate the kinds of disruption risk management theories described in [89, 93], which enable Waffle House Restaurants to succeed in effectively responding to hurricanes.

4.4.2 Decision Tradeoffs

There are several decisions that must be made before or during a hurricane response that highlight the tradeoffs faced by functional groups within a single organization or multiple stakeholders in a collaborative response. This case discusses many interactions between functional groups and highlights collaboration between Waffle House Restaurants, its suppliers, the National Guard, and other stakeholders during a hurricane response. To illustrate the decision processes involved in such relationships and describe the collaborative activities in the Waffle House Restaurants response plan, we consider two decisions that involve tradeoffs: the choices of hurricane menu items and store reopening.

The choice of items on the hurricane menu highlights the tradeoffs between customer satisfaction and efficient operation with limited resources. Customer preference and the time and space required for food preparation are important factors in the menu decision, particularly from the perspective of the operations functional group. From the perspective of the purchasing group, supplier relationships and capability to sustain deliveries also impact menu choices. However, from the control perspective, the storage and refrigeration requirements and item shelf life impact this decision. The choice of menu items impacts suppliers as well. Waffle House Restaurants have managed this relationship by establishing clear communication processes with suppliers and an ordering timeline for items that are specific to hurricane response, including greater quantities of ham and canned beverages instead of fountain supplies. Limited resources may require reduced offerings of goods or services across most organizations, but the best ways to manage this depend on the objectives and functions of the organization.

Many factors also affect a store reopening decision. Proximity to a major roadway or airport are positive factors from the operations standpoint for ease of customer access, from the perspective of construction and equipment personnel who need to supply gasoline for generators, and for the purchasing group to ensure that supplies can get in quickly. The current inventory levels at the candidate stores and their location with respect to other stores from which inventory can be transferred are important factors. If one store has electricity and the other does not, this has a strong influence on the reopening decision. If generators or food supplies are scarce, the order of importance of the factors can also change.

4.4.3 Generalizing Lessons Learned

Waffle House Restaurants has learned from years of experience responding to hurricanes that impacted their locations. Other organizations can benefit from these lessons, including the need for well-documented processes and continuous updating based on new experiences. The hurricane response process is a good illustration of the phases of crisis management. Adopting a critical supply chain management component, the company has developed relationships with suppliers to help accomplish its goal of reopening stores as quickly as possible. Ideas such as the hurricane menu and the hurricane inventory sheets with par levels are transferable to other organizations that must make decisions about the breadth of goods and services to offer in a disaster's aftermath and on their inventory strategies. Pre-positioning inventory and drawing from nearby stores that are unable to open has also improved Waffle House Restaurants' response efforts and could be reapplied in other industries, government agencies [60], or NGOs. Perhaps most importantly, Waffle House Restaurants has a proactive mentality that extends through all levels of the organization, stemming from the company's founding philosophy to focus on the well-being of their employees and customers.

The private sector, governmental agencies, and non-governmental organizations can learn from the application of crisis and supply chain management methodologies in the example of Waffle House Restaurants. Key take-away messages for others seeking to improve their effectiveness are to develop the practices described by Weick and Sutcliffe [103], to understand the important phases of crisis management [76], and to implement supply chain management principles with the awareness that planning for the disruption risks posed by disaster response may require different models than day-to-day supply chain practices.

4.5 Conclusions and Future Research

Waffle House Restaurants' hurricane response processes have developed over decades of experience. The company's commitment to its associates and to the communities in which they operate has driven the continuous improvement in these processes. Each functional area of the organization has clear responsibilities and plays a key role in enabling quick recovery following even a major event such as Hurricane Katrina. As a result, Waffle House Restaurants have earned recognition as a leader in disaster relief. The lessons learned continue to shape the future of Waffle House Restaurants and provide insight for other organizations to further develop their own response capabilities.

This exploratory case study aimed to identify a private sector organization that succeeds in disaster response, describe the crisis and supply chain management tools it employs, understand the roles of functional groups and collaboration in the organization's processes, and extend lessons to other organizations. The results of this study raise several questions for future research. Additional studies should be done to determine whether organizations that employ similar crisis and supply chain management practices meet with similar success in disaster response. Comparisons can be made between U.S. and international organizations, as well as between private sector, government, and non-governmental organizations. The development of supply chain management models and tools that can further improve disaster response, especially those that enable organizations to assess risk, resilience, and response approaches quantitatively, is also an important area of future research.

CHAPTER V

CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

The challenges inherent in providing goods and services to those affected by humanitarian and public health emergencies require advanced logistics solutions. This thesis adds to the understanding of decentralized networks and the methods used to study them, particularly in the humanitarian context, and characterizes qualities of a successful disaster response supply chain. Studying these supply chains contributes to theory and practice in ways that can also benefit other application domains. The work of this thesis leads to a number of future research avenues, some of which we summarize here.

The contributions of this thesis point to opportunities for future work on the theory of decentralized systems. We proved that an equilibrium solution to the problem introduced in Chapter 2 can be found efficiently. However, there are frequently multiple equilibria in the system. Developing methods for finding, characterizing the performance of, and improving the set of equilibrium solutions remains a potentially fruitful area of research. We briefly discuss a computational approach for finding the best or worst equilibrium solution in Appendix B, but understanding the theoretical properties of the set of equilibrium solutions in a variety of network structures could lead to improved methods of managing decentralized systems. Furthermore, work in this area may build upon the idea of equilibrium-obtaining congestion weight vectors, which guarantee that the central optimal solution is *one* possible equilibrium, and lead to the development of robust mechanisms under which *no* equilibrium solution performs too poorly.

In the decentralized systems described in this thesis, individuals make choices among facilities based on objectives that incorporate travel time, congestion, and facility-specific weights on congestion. Additional individual objectives, and the resulting decentralized system performance, are of interest for future research. Different functional forms as well as individual-specific objective parameters may be useful for modeling other scenarios arising in practice. Eliciting individual choice patterns through case studies of actual systems or behavioral surveys could provide further insight to incorporate into the decentralized models. This line of research may lead to the design of additional mechanisms, akin to the equilibrium-obtaining congestion weight vectors, that can improve decentralized system performance.

The first and second parts of the thesis considered decentralized logistics systems in which facility locations were fixed and pre-specified. In practice, many organizations face choices about facility location and capacity, whether in the design of new logistics systems or when expanding existing ones. Most current facility location models adopt a centralized perspective or, if individual decisions are considered, assume that distance is the primary criterion. Expanding the decentralized modeling framework described in this thesis to include facility location and capacity decisions, therefore, is an important area of future research. As one example, this approach could be used to design a distribution system such as the one discussed in Chapter 3 in a way that minimizes inequities in access.

Immediate next steps related to the study of accessibility and equity were described in Chapter 3 and include the expansion of the study to the Southeast region of the United States, to additional regions, and to a product specifically intended for children. This work will facilitate comparison between regions and lead to policy recommendations that take into account specific features of different areas. Additional future work in this area could examine dynamics in the system over time. The current approach uses the entire quantity of product available during the shortage period, but shipments to service locations actually took place over time. The distribution process also continued beyond the shortage period. Incorporating the temporal aspect of the problem and examining the differences between the shortage and surplus phases may provide additional information for improving future distribution efforts.

The third part of this thesis examined Waffle House Restaurants' successful hurricane response efforts. This study highlighted the practices of a single organization, but in many humanitarian contexts multiple agencies interact. Further studies could characterize effective collaborations between organizations in practice and develop decision support tools to increase the impact that these partnerships have through their operations.

APPENDIX A

PROOFS OF CHAPTER 2 RESULTS

Proof of Theorem 3. The proof consists of two parts. We first demonstrate the lower bound by providing a network structure for which the lower bound is tight for any value of $\alpha_{max} \geq 1$. We then prove the upper bound.

Consider the network illustrated in Figure 2, in which the number of customers, n, is equal to the number of facilities, m. The distance from each customer to the facility located in the clockwise direction is 0, while that to the facility in the counter-clockwise direction is α_{max}. The congestion weight associated with each facility is α_{max}. The optimal solution is shown on the left; all users travel in a clockwise direction and the total cost is n. The cost perceived by each individual is α_{max}. However, the solution on the right is also an equilibrium solution in which each user incurs travel time α_{max} and congestion 1, resulting in a total cost of $n\alpha_{max} + n$. This solution is an equilibrium, because the cost perceived at the current solution for each individual is equal to the perceived cost of switching: $2\alpha_{max}$. Therefore, for networks with this structure for any $n = m$, we have

$$\text{Price of Anarchy} = \frac{n\alpha_{max} + n}{n} = \alpha_{max} + 1,$$

providing a tight lower bound on the price of anarchy.

We now prove the upper bound. Consider an equilibrium solution x and a central optimal solution x^*. Let d_{ij} be the distance from customer i to facility j, $I(j)$ the set of customers served at j in the equilibrium solution, $I^*(j)$ the set of customers served at j in the central optimal solution, and x_j and x_j^* the cardinality of $I(j)$ and $I^*(j)$, respectively. We will make use of the following relationship in our proof, adapted from [8]:

$$\sum_j \sum_{i \in I(j)} d_{ij} + \sum_j x_j^*(x_j + 1) \le 2.5 \left(\sum_j \sum_{i \in I^*(j)} d_{ij} + \sum_j x_j^{*2} \right) \tag{50}$$

The equilibrium solution x satisfies the Equilibrium Condition for each customer i, which is given by $d_{ij_i} + \alpha_{j_i} x_{j_i} \le d_{ij_i^*} + \alpha_{j_i^*}(x_{j_i^*} + 1)$. Here j_i is the facility at which i is served in the equilibrium solution and j_i^* that in the central optimal solution. Aggregating this expression over all customers i, we obtain

$$\sum_i (d_{ij_i} + \alpha_{j_i} x_{j_i}) \le \sum_i \left(d_{ij_i^*} + \alpha_{j_i^*}(x_{j_i^*} + 1) \right). \tag{51}$$

Regrouping by facilities gives

$$\sum_j \sum_{i \in I(j)} (d_{ij} + \alpha_j x_j) \le \sum_j \sum_{i \in I^*(j)} (d_{ij} + \alpha_j (x_j + 1)). \tag{52}$$

Simplification yields

$$\sum_j \sum_{i \in I(j)} d_{ij} + \sum_j \alpha_j x_j^2 \le \sum_j \sum_{i \in I^*(j)} d_{ij} + \sum_j \left(\alpha_j x_j^*(x_j + 1) \right). \tag{53}$$

Focusing on the expression on the right of (53), observe that

$$\sum_j \sum_{i \in I^*(j)} d_{ij} + \sum_j \left(\alpha_j x_j^*(x_j + 1) \right) \le \alpha_{max} \left(\sum_j \sum_{i \in I^*(j)} d_{ij} + \sum_j x_j^*(x_j + 1) \right), \tag{54}$$

since $\alpha_{max} \ge \alpha_j \ge 1$ for all j. The expression in parentheses on the right of (54) is equal to that on the left of (50). By combining (50), (53), and (54), we obtain

$$\sum_j \sum_{i \in I(j)} d_{ij} + \sum_j \alpha_j x_j^2 \le 2.5\alpha_{max} \left(\sum_j \sum_{i \in I^*(j)} d_{ij} + \sum_j x_j^{*2} \right). \tag{55}$$

Note that the expression in parentheses on the right is the cost of the central optimal solution, so all that remains is to relate the expression on the left of (55) to the true cost of the equilibrium solution. That is straightforward since $\alpha_j \ge 1$ for all j, and we are left with

$$\sum_j \sum_{i \in I(j)} d_{ij} + \sum_j x_j^2 \le \sum_j \sum_{i \in I(j)} d_{ij} + \sum_j \alpha_j x_j^2 \le 2.5\alpha_{max} \left(\sum_j \sum_{i \in I^*(j)} d_{ij} + \sum_j x_j^{*2} \right), \tag{56}$$

as desired. □

Proof of Lemma 1. First observe that only the congestion at facilities j and k and the travel time of customer i are changed by this move. In solution x, these values are given by

$$(x_j + 1)^2 + x_k^2 + d_{ij}.$$

Similarly, following i's move, the relevant congestion and travel time terms become

$$x_j^2 + (x_k + 1)^2 + d_{ik}.$$

Expanding terms and subtracting the cost in the original solution from the cost after the move, we obtain

$$\left(x_j^2 + x_k^2 + 2x_k + 1 + d_{ik}\right) - \left(x_j^2 + 2x_j + 1 + x_k^2 + d_{ij}\right) = 2x_k - 2x_j + d_{ik} - d_{ij}.\square$$

Proof of Lemma 2. Let $\bar{x}$ be a solution in which $\bar{x}_{ij} = 1$, $d_{ij} \leq d_{ik}$ for all $k \neq j$, and

$$d_{ij} + \sum_{p=1}^{n} \bar{x}_{pj} > d_{ik} + \sum_{p=1}^{n} \bar{x}_{pk} + 1$$

for at least one k. In words, $\bar{x}$ satisfies i's Equilibrium Condition when $\alpha = 0$ but not when $\alpha = 1$. Let $\tilde{x}$ be a solution in which $\bar{x}_{pj} = \tilde{x}_{pj}$ for all $p \neq i$, $\tilde{x}_{ik} = 1$, and

$$d_{ik} + \sum_{p=1}^{n} \tilde{x}_{pk} \leq d_{ij} + \sum_{p=1}^{n} \tilde{x}_{pj} + 1$$

for all $j \neq k$. In words, $\tilde{x}$ differs from $\bar{x}$ only in the assignment of customer i, and $\tilde{x}$ satisfies i's Equilibrium Condition when $\alpha = 1$ but may or may not satisfy it when $\alpha = 0$. Then the total cost of $\tilde{x}$ is no greater than that of $\bar{x}$:

$$\sum_{j=1}^{m} \sum_{i=1}^{n} d_{ij}\tilde{x}_{ij} + \sum_{j=1}^{m} \left(\sum_{i=1}^{n} \tilde{x}_{ij}^2 \right) < \sum_{j=1}^{m} \sum_{i=1}^{n} d_{ij}\bar{x}_{ij} + \sum_{j=1}^{m} \left(\sum_{i=1}^{n} \bar{x}_{ij}^2 \right).$$

Using the notation of Lemma 1, let $\bar{x}_j$ be the total number of customers at j in solution $\bar{x}$ excluding i. Similarly, let $\bar{x}_k$ be the total number of customers at k. Then

$d_{ij} - d_{ik} + \bar{x}_j - \bar{x}_k > 0$ because $\bar{x}$ does not satisfy i's Equilibrium Condition when $\alpha = 1$. Since $d_{ij} \leq d_{ik}$, this implies that $\bar{x_j} > \bar{x_k}$. Together, these two inequalities imply that $2\bar{x}_k - 2\bar{x}_j + d_{ik} - d_{ij} < 0$. As shown in the proof of Lemma 1, this expression is precisely the difference in system cost between $\bar{x}$ and $\tilde{x}$, proving that moving customer i from facility j to facility k results in a net decrease in system cost. □

Proof of Theorem 6. The proof consists of two parts. First, we begin at an arbitrary equilibrium solution under the $\alpha = 0$ framework and move customers one by one (possibly moving a customer more than once) such that the current move satisfies that customer's Equilibrium Condition under the $\alpha = 1$ framework. We show that the cumulative change in system cost that results from these moves is strictly negative. Second, we show that this iterative process reaches an equilibrium solution under the latter framework in a pseudo-polynomial number of steps under the assumption that travel times and α values are rational numbers.

To simplify presentation, let

$$x_j := \sum_{i=1}^{n} x_{ij} \quad \text{for each } j.$$

Consider an arbitrary customer i and the sequence of iterations $q = 0, 1, 2, ..., Q$ in which i is the customer that is moved. (Note that any number of moves may occur by other customers between consecutive moves by i.) Let x_j^q be the number of customers at facility j in i's qth iteration, for all j and for $q = 0, 1, 2, ..., Q$. Let o_q be the facility at which i is served prior to move q and d_q be that after the move. From the Equilibrium Condition with $\alpha = 1$, we know

$$x_{d_0}^0 - x_{o_0}^0 + d_{id_0} - d_{io_0} < 0, \text{or equivalently, } x_{o_0}^0 - x_{d_0}^0 + d_{io_0} - d_{id_0} > 0. \tag{57}$$

From Lemma 2, we also know that i's first move improves the system cost, with the net change given by

$$2x_{o_0}^0 - 2x_{d_0}^0 + d_{io_0} - d_{id_0} > 0. \tag{58}$$

Furthermore, we know from the Equilibrium Condition with $\alpha = 1$ that

$$x^q_{d_q} - x^q_{o_q} + d_{id_q} - d_{io_q} < 0 \tag{59}$$

in any iteration q in which i moves from o_q to d_q. Summing (59) over all of i's moves, we obtain

$$\sum_{q=1}^{Q} (x^q_{d_q} - x^q_{o_q} + d_{id_q} - d_{io_q}) < 0. \tag{60}$$

Since $d_{io_q} = d_{id_{q-1}}$, travel time terms cancel except for those to the first and last facilities. We thus obtain

$$\sum_{q=1}^{Q} (x^q_{d_q} - x^q_{o_q} + d_{id_q} - d_{io_q}) = \sum_{q=1}^{Q} (x^q_{d_q} - x^q_{o_q}) + d_{id_Q} - d_{id_0} < 0. \tag{61}$$

Next observe that

$$x^0_{o_0} - x^0_{d_0} > d_{id_0} - d_{io_0} \geq d_{id_0} - d_{id_Q} > \sum_{q=1}^{Q} (x^q_{d_q} - x^q_{o_q}), \tag{62}$$

where the leftmost inequality follows from (57), the middle inequality from the fact that we started from an equilibrium solution under $\alpha = 0$ implying that $d_{o_0} \leq d_{d_Q}$, and the final inequality from (61). We now subtract (57) from (61) to obtain

$$\sum_{q=1}^{Q} (x^q_{d_q} - x^q_{o_q}) + d_{id_Q} - d_{id_0} - (x^0_{o_0} - x^0_{d_0} + d_{io_0} - d_{id_0}) < 0. \tag{63}$$

Next observe

$$\begin{aligned} &\left(\sum_{q=1}^{Q} (x^q_{d_q} - x^q_{o_q}) - (x^0_{o_0} - x^0_{d_0}) \right) \\ &+ \left(\sum_{q=1}^{Q} (x^q_{d_q} - x^q_{o_q}) + d_{id_Q} - d_{id_0} - (x^0_{o_0} - x^0_{d_0} + d_{io_0} - d_{id_0}) \right) < 0, \end{aligned} \tag{64}$$

where the first group of terms is less than 0 from (62) and the second from (63). Regrouping (64) we obtain

$$\left(\sum_{q=1}^{Q} (2x^q_{d_q} - 2x^q_{o_q}) + d_{id_Q} - d_{id_0} \right) - \left((2x^0_{o_0} - 2x^0_{d_0}) + d_{io_0} - d_{id_0} \right) < 0. \tag{65}$$

From Lemma 1, we see that the group of terms on the right gives the change in system cost as a result of i's first move while that on the left is the cumulative change resulting from moves $1, 2, ..., Q$. Since this cumulative change is strictly negative, we have shown that the system cost decreases as a result of a sequence of moves by an arbitrary customer, without regard to the moves by other customers. Since this is true for each customer, it follows that the system cost decreases as a result of the combined moves of all customers. All that remains to be shown is that iteratively moving customers from an equilibrium solution under the $\alpha = 0$ framework will achieve an equilibrium solution under the $\alpha = 1$ framework.

To prove this, we rely on knowledge of properties of the potential function of congestion games described by Rosenthal [79]. He showed that every minima of the potential function is a Nash equilibrium (in our terms, an equilibrium under $\alpha = 1$). Moreover, every selfish move by any customer results in a decrease in the potential function (even if the system objective increases). In fact, the decrease in the potential function is equal to the decrease that customer achieves in his own objective function. Since the potential function is non-negative and since it strictly decreases with every selfish move by every customer, we will certainly reach a potential function minima and thus an equilibrium solution under the $\alpha = 1$ framework by iteratively moving from any equilibrium solution under the $\alpha = 0$ framework in this way. Note that we may reach an equilibrium under $\alpha = 1$ before we find a potential function minima; all minima are equilibrium solutions under $\alpha = 1$ but the converse is not true. To see that the process requires a pseudo-polynomial number of moves when travel times and α values are rational, observe that multiplying all arc costs by the same suitably large number in both the original and minimum cost flow networks from Theorem 1 preserves the potential function transformation. After this scaling, every selfish move decreases the potential function value by at least 1, so the number of moves needed to reach the potential function minimum is pseudo-polynomial.

Finally, since we start at an arbitrary equilibrium solution $x(\alpha = 0)$ under the $\alpha = 0$ framework and find an equilibrium solution $x(\alpha = 1)$ under the $\alpha = 1$ framework with no greater cost, the result holds when we start at $x^*(\alpha = 0)$, the equilibrium solution under the former framework with lowest total system cost. Thus

$$z^*(\alpha = 1) \leq z(\alpha = 1) \leq z^*(\alpha = 0).\square$$

Proof of Theorem 7. Clearly the 0 vector is a feasible solution to D'. It remains to be shown that this is a unique solution. For contradiction, suppose there is a feasible solution y' with at least one component strictly greater than 0.

Divide the j^{th} constraint of D' (6) by x_j^* and introduce surplus variable s_j to obtain the equivalent expression

$$\frac{x_j^*}{x_j^* + 1} \sum_k y_{kj} - \sum_k y_{jk} + s_j = 0 \qquad \forall\ j. \tag{66}$$

Note that each $s_j \leq 0$ in any feasible solution since the original constraints (6) are all greater than or equal to 0. Let $\lambda_j = \frac{x_j^*}{x_j^*+1}$ and note that $\lambda_j < 1$ for all j. Aggregating constraints over all j yields

$$\sum_j \sum_k \lambda_j y_{kj} - \sum_j \sum_k y_{jk} + \sum_j s_j = 0. \tag{67}$$

Recognizing that each y_{kj} occurs once in the first term of (67) and once in the second, regroup:

$$\sum_j \sum_k (\lambda_j - 1) y_{jk} + \sum_j s_j = 0. \tag{68}$$

Now since $\lambda_j < 1$ and $y_{kj} \geq 0$, we know that the term on the left is less than or equal to zero, with equality only possible when $y_{kj} = 0$ for all j, k. This implies that $\sum_j s_j \geq 0$ and that if any $y_{kj} > 0$, then $\sum_j s_j > 0$. However, our proposed solution y', with at least one non-zero component, clearly violates the original dual constraints since $s_j \leq 0$ in all feasible dual solutions. This contradiction implies that the unique feasible solution to D' has $y_{kj} = 0$ for all k, j.

Since the dual problem D' is both feasible and bounded, the primal problem P' always has a feasible solution. □

Proof of Lemma 4. The proof consists of two parts. In the first part, we show that there are no negative cost cycles consisting of original dual nodes and arcs. In the second part, we prove that even if there is a negative cost cycle when flow loss factors are included, such a cycle is not included in any most-negative $S-T$ path in the dual network.

First we examine the dual network without considering flow loss factors. For contradiction, assume there exists a negative cost cycle consisting of c original dual nodes and arcs. Without loss of generality, let these be nodes numbered $1, \ldots, c$. The sum of the arc costs on this path is given by $C = D_{12} + D_{23} + \ldots + D_{c-1,c} + D_{c1}$, which is assumed to be negative. Recall that $D_{kj} = d_{ik} - d_{ij}$ for some individual i served at facility j in the central optimal solution. This implies that $C = d_{i_{12}1} - d_{i_{12}2} + d_{i_{23}2} - d_{i_{23}3} + \ldots + d_{i_{c-1,c}c-1} - d_{i_{c-1,c}c} + d_{i_{c1}c} - d_{i_{c1}1} < 0$, where i_{jk} represents the individual from which the value D_{jk} is obtained. In each case, this individual is currently served at facility k. Observe, however, that the expression C also represents the net change in travel time incurred when every individual i_{jk} switches to facility j. The total system congestion is unchanged by this switch since one individual is removed and another added at each facility. Since $C < 0$, the proposed switch results in a decrease in the cost of the central optimal solution and contradicts its optimality. Therefore, no negative cost cycles exist if flow losses are ignored.

We will now show that, despite the existence of negative cost cycles when flow loss factors are included, such a cycle is not part of any most-negative $S-T$ path in the dual network. For contradiction, assume there exists a negative cost cycle consisting of c original dual nodes and arcs when flow loss factors are included. Without loss of generality, let these be nodes numbered $1, \ldots, c$. Furthermore, assume that this cycle is part of a most-negative $S-T$ path, that is, $(S, 1, \ldots, c, 1, T)$ is the most negative path connecting the source, S, to the sink, T. We have just shown that

$$D_{12} + D_{23} + \ldots + D_{c-1,c} + D_{c1} \geq 0. \tag{69}$$

Recall that arcs leading from the source to each facility node and from each facility node to the sink all have cost 0. The total cost of the $S - T$ path is then given by the total cost of the cycle, which is assumed to be negative when flow losses are considered:

$$D_{12} + \lambda_2 D_{23} + \ldots + \prod_{j=2}^{c-1} \lambda_j D_{c-1,c} + \prod_{j=2}^{c} \lambda_j D_{c1} < 0. \tag{70}$$

Moreover, since this is assumed to be the most negative $S-T$ path, every subpath in the cycle that terminates at node 1 must also have negative cost when flow losses are considered. (Otherwise, going directly to the sink would result in lower cost.) This observation is represented mathematically as follows:

$$\lambda_2 D_{23} + \lambda_2 \lambda_3 D_{34} + \ldots + \prod_{j=2}^{c-1} \lambda_j D_{c-1,c} + \prod_{j=2}^{c} \lambda_j D_{c1} < 0 \tag{71}$$

$$\lambda_2 \lambda_3 D_{34} + \lambda_2 \lambda_3 \lambda_4 D_{45} + \ldots + \prod_{j=2}^{c-1} \lambda_j D_{c-1,c} + \prod_{j=2}^{c} \lambda_j D_{c1} < 0 \tag{72}$$

$$\ldots \tag{73}$$

$$\prod_{j=2}^{c} \lambda_j D_{c1} < 0 \tag{74}$$

Recall that $0 < \lambda_j < 1$ for all j. Factoring out the common λ_j terms from each of the expressions (71)-(74), it is apparent that each of the terms in parentheses below is negative:

$$\lambda_2 \left(D_{23} + \lambda_3 D_{34} + \ldots + \prod_{j=3}^{c-1} \lambda_j D_{c-1,c} + \prod_{j=3}^{c} \lambda_j D_{c1} \right) < 0 \tag{75}$$

$$\lambda_2\lambda_3\left(D_{34}+\lambda_4 D_{45}+\ldots+\prod_{j=4}^{c-1}\lambda_j D_{c-1,c}+\prod_{j=4}^{c}\lambda_j D_{c1}\right)<0 \quad (76)$$

$$\ldots \quad (77)$$

$$\prod_{j=2}^{c}\lambda_j\left(D_{c1}\right)<0 \quad (78)$$

Combining all of these observations, we obtain:

$$0>D_{12}+\lambda_2\left(D_{23}+\lambda_3 D_{34}+\ldots+\prod_{i=3}^{c-1}\lambda_i D_{c-1,c}+\prod_{i=3}^{c}\lambda_i D_{c1}\right) \quad (79)$$

$$>D_{12}+D_{23}+\lambda_3\left(D_{34}+\ldots+\prod_{i=4}^{c-1}\lambda_i D_{c-1,c}+\prod_{i=4}^{c}\lambda_i D_{c1}\right) \quad (80)$$

$$\ldots \quad (81)$$

$$>\ D_{12}+D_{23}+D_{34}+\ldots+\lambda_{c-1}\left(D_{c-1,c}+\lambda_c D_{c1}\right)>\ D_{12}+D_{23}+D_{34}+\ldots+D_{c-1,c}+\lambda_c D_{c1} \quad (82)$$

$$>\ D_{12}+D_{23}+D_{34}+\ldots+D_{c-1,c}+D_{c1}\geq\ 0, \quad (83)$$

where the first inequality follows from (70), the intermediate inequalities from (75)-(78) and the fact that λ_j's are between 0 and 1, and the final inequality from (69). However, this is clearly a contradiction. Therefore, the proposed negative cycle can never be a most-negative $S-T$ path. □

Proof of Theorem 10. We know that primal optimal value is equal to the value of the minimum cost circulation in the dual, given by

$$-\alpha_j=\frac{1}{x_j^*+1}\cdot C_{p_{min}}.$$

To bound the dual optimal value, we introduce two definitions. Let $D_{min}=min_{k,j}\{D_{kj}\}$ and let $\lambda_{max}=max_j\{\frac{x_j^*}{x_j^*+1}\}$. These values capture the arc with least cost and the

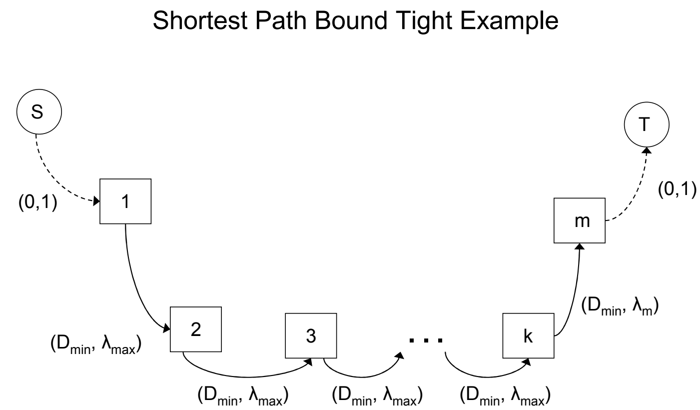


Figure 22: Tight Example for Single α Bound.

arc on which there is the smallest loss of flow, respectively. The maximum number of arcs with non-zero cost in an $S-T$ path is $m-1$ due to Lemma 4. Together, this information implies the following bound on the minimum cost $S-T$ path:

$$C_{p_{min}} \geq D_{min} \cdot (1 + \lambda_{max} + \lambda_{max}^2 + \ldots \lambda_{max}^{m-3} + \lambda_{max}^{m-2}) = D_{min} \left(\sum_{r=0}^{m-2} \lambda_{max}^r \right).$$

Substituting the expression for the sum of a geometric series gives an exact expression for the bound in terms of λ_{max}, m, and D_{min}:

$$C_{p_{min}} \geq D_{min} \left(\sum_{r=0}^{m-2} \lambda_{max}^r \right) = D_{min} \left(\frac{1 - \lambda_{max}^{m-1}}{1 - \lambda_{max}} \right).$$

By combining this path cost with the original flow value on the path and dividing by -1 to isolate α_j, we obtain the desired result:

$$\alpha_j = \leq -\frac{1}{x_j^* + 1} D_{min} \left(\frac{1 - \lambda_{max}^{m-1}}{1 - \lambda_{max}} \right).$$

A tight example can be constructed for any number of facilities, m, as illustrated in Figure 22. Suppose, without loss of generality, that we seek the minimum feasible value of α_1. An $S-1-\ldots-T$ path that visits all facilities, with a cost of D_{min} on all original network arcs and a loss factor of λ_{max} on the first $m-2$ such arcs, achieves the bound. $\square$

Proof of Theorem 11. The proof is similar to that of Theorem 10. In this case, the value of the minimum cost circulation is given by the sum of the values of the shortest paths associated with each facility. This observation follows from Theorem 9 and from the fact that there are no upper bounds on flow along the original dual arcs. As a result, flow along arc (S, j) continues along the shortest path from j to T for each j and the minimum cost circulation is simply a composite of m flows along the respective shortest paths. We will again bound the primal optimal value using information about the dual problem.

We know that the primal and dual optimal values are equal:

$$-\sum_j \alpha_j = \sum_j \left(\frac{1}{x_j^* + 1} \cdot C_{p^j_{min}} \right).$$

Here p^j_{min} denotes the shortest path in the dual network connecting facility j to T and $C^j_{p_{min}}$ its cost. Define $\lambda_{min} = min_j\{\frac{x_j^*}{x_j^*+1}\}$. Examining the dual optimal expression on the right-hand side, we have

$$\sum_j \left(\frac{1}{x_j^* + 1} \cdot C_{p^j_{min}} \right) = \sum_j \left(1 - \frac{x_j^*}{x_j^* + 1} \right) \cdot C_{p^j_{min}} = \sum_j (1 - \lambda_j) \cdot C^j_{p_{min}} \geq (1 - \lambda_{min}) \sum_j C^j_{p_{min}},$$

where the final inequality follows from the facts that $C^j_{p_{min}} \leq 0$ and $(1 - \lambda_j) \leq (1 - \lambda_{min})$ for all j.

We next bound the sum of the shortest path costs. As in the proof of Theorem 10, let $\lambda_{max} = max_j\{\frac{x_j^*}{x_j^*+1}\}$ and $D_{min} = min_{k,j}\{D_{kj}\}$. These values capture the arc with least cost and the arc on which there is the smallest loss of flow, respectively. The fundamental idea of the proof of Theorem 10 is to bound the value of a single path with the most negative arc costs, the greatest number of arcs, and the least flow loss. Recall, however, that this bounding path visits each facility. By basic properties of shortest paths, the part of this bounding path that connects a particular facility, j, to T is also a shortest path from j to T. The sum of the costs of all shortest paths can thus be bounded by adding the costs of each $j - T$ segment of the single bounding path.

$$\sum_j C^j_{p_{min}} \geq D_{min} \cdot \left(\frac{1-\lambda_{max}^{m-1}}{1-\lambda_{max}} + \frac{1-\lambda_{max}^{m-2}}{1-\lambda_{max}} + \ldots + \frac{1-\lambda_{max}^{2}}{1-\lambda_{max}} + \frac{1-\lambda_{max}}{1-\lambda_{max}} \right)$$

$$= D_{min} \sum_{r=1}^{m-1} \left(\frac{1-\lambda_{max}^{r}}{1-\lambda_{max}} \right).$$

Simplifying the summation, we obtain

$$\sum_{r=1}^{m-1} \left(\frac{1-\lambda_{max}^{r}}{1-\lambda_{max}} \right) = \left(\frac{(m-1) - (-1 + 1 + \lambda_{max} + \lambda_{max}^2 + \ldots + \lambda_{max}^{m-1})}{1-\lambda_{max}} \right)$$

$$= \frac{1}{1-\lambda_{max}} \cdot \left(m - \sum_{r=0}^{m-1} \lambda_{max}^r \right) = \frac{1}{1-\lambda_{max}} \cdot \left(m - \frac{1-\lambda_{max}^m}{1-\lambda_{max}} \right).$$

Finally, combining all of these facts and dividing by -1 to isolate the sum of the α_j values, we obtain

$$\sum_j \alpha_j = -\sum_j \left(\frac{1}{x_j^* + 1} \cdot C_{p^j_{min}} \right) \leq -\frac{1-\lambda_{min}}{1-\lambda_{max}} \cdot D_{min} \cdot \left(m - \frac{1-\lambda_{max}^m}{1-\lambda_{max}} \right)$$

$$= -\frac{x_{max}^* + 1}{x_{min}^* + 1} \cdot D_{min} \cdot \left(m - \frac{1-\lambda_{max}^m}{1-\lambda_{max}} \right).$$

A network analogous to that in Figure 22, with additional arcs (S, j) for all j and $\lambda_{min} = \lambda_{max}$, demonstrates that this bound is tight. $\square$

APPENDIX B

CHAPTER 2 SUPPLEMENTARY MATERIAL

This appendix contains supplementary material describing our early research results on the topics described in Chapter 2.

B.1 Some Special Cases of the Decentralized Problem

This section describes several results for specific cases of the Facility-Specific Congestion Weights Problem. We first present a description of networks for which we have proven that the price of anarchy is equal to 2 when $\alpha_j = 1$ for all j. These are followed by results for special network structures in the case that congestion weights α_j are 0 for all facilities j.

B.1.1 When Congestion Weights Equal 1 for All Facilities

Recall from Corollary 2 that the price of anarchy when $\alpha_j = 1$ for all j is at least 2. There are a number of special cases for which the price of anarchy is exactly 2. Before presenting these cases, we prove a useful relationship between central optimal solution and decentralized solutions with α's equal to 1.

Lemma 5 *Let d_{ij} be the distance from customer i to facility j, $I(j)$ the set of customers served at j in a decentralized solution with $\alpha_j = 1$ for all j, $I^*(j)$ the set of customers served at j in the central optimal solution, and x_j and x_j^* the cardinality of $I(j)$ and $I^*(j)$, respectively. Then*

$$\sum_j \sum_{i \in I^*(j)} d_{ij} + \sum_j x_j^{*2} \leq \sum_j \sum_{i \in I(j)} d_{ij} + \sum_j x_j^2 \leq \sum_j \sum_{i \in I^*(j)} d_{ij} + \sum_j \left(x_j x_j^* + x_j^* \right).$$

Proof: Begin by considering the Equilibrium Condition for each customer i. This can be written as

$$d_{ij_i} + x_{j_i} \leq d_{ij_i^*} + x_{j_i^*} + 1,$$

where j_i is the facility at which i is served in the decentralized solution and j_i^* that in the central optimal solution. Aggregating this expression over all customers i, we obtain

$$\sum_i \left(d_{ij_i} + x_{j_i}\right) \leq \sum_i \left(d_{ij_i^*} + x_{j_i^*} + 1\right).$$

Regrouping by facilities gives

$$\sum_j \sum_{i \in I(j)} \left(d_{ij} + x_j\right) \leq \sum_j \sum_{i \in I^*(j)} \left(d_{ij} + x_j + 1\right).$$

Simplification yields

$$\sum_j \sum_{i \in I(j)} d_{ij} + \sum_j x_j^2 \leq \sum_j \sum_{i \in I^*(j)} d_{ij} + \sum_j \left(x_j x_j^* + x_j^*\right).$$

The expression on the left gives the total system cost of the decentralized solution, which is at least as much as that of a central optimal solution. This gives

$$\sum_j \sum_{i \in I^*(j)} d_{ij} + \sum_j x_j^{*2} \leq \sum_j \sum_{i \in I(j)} d_{ij} + \sum_j x_j^2 \leq \sum_j \sum_{i \in I^*(j)} d_{ij} + \sum_j \left(x_j x_j^* + x_j^*\right),$$

as desired. □

Lemma 6 *Let* $x_j = \sum_{i=1}^n x_{ij}$ *for each* j*, the total congestion at facility* j *in a decentralized solution with* $\alpha_j = 1$ *for all* j*. Similarly, let* $x_j^* = \sum_{i=1}^n x_{ij}^*$ *for each* j*, the total congestion at facility* j *in the central optimal solution. If* $x_j = x_j^*$ *for every* $j = 1, \ldots, m$*, then the price of anarchy is 2; that is*

$$\frac{z(\alpha = 1)}{z^*} = 2.$$

Proof: We make use of the result in Lemma 5. Substituting $x_j = x_j^*$ for all j into the rightmost expression gives

$$\sum_j \sum_{i \in I(j)} d_{ij} + \sum_j x_j^2 \leq \sum_j \sum_{i \in I^*(j)} d_{ij} + \sum_j \left(x_j^{*2} + x_j^*\right) \leq 2 \cdot \left(\sum_j \sum_{i \in I^*(j)} d_{ij} + \sum_j x_j^{*2}\right).$$

$\square$ Moreover, the network in Figure 2 with $\alpha_j = 1$ for all j shows that the bound is tight. $\square$

Lemma 7 *If the central optimal solution splits customers equally between facilities, that is, if* $x_j^* = \frac{n}{m}$ *for all* j*, then*

$$\frac{z(\alpha = 1)}{z^*} = 2.$$

Proof: It suffices to show that $\sum_j \left(x_j x_j^* + x_j^* \right) \leq 2 \sum_j x_j^{*2}$ and to substitute this value into the expression from Lemma 5. Letting $x_j^* = \frac{n}{m}$ for all j, we have

$$\sum_j \left(x_j x_j^* + x_j^* \right) = \frac{n^2}{m} + n \leq 2 \cdot \frac{n^2}{m} = 2 \cdot \sum_j x_j^{*2}.$$

The middle inequality holds whenever $m \leq n$, which must be the case since each facility has exactly $\frac{n}{m}$ customers and customers are integers. Again, the network in Figure 2 shows that the bound is tight. $\square$

Lemma 8 *If* $m = 2$ *and* $n = 2$*, that is, if there are exactly two facilities and two customers in the network, then*

$$\frac{z(\alpha = 1)}{z^*} = 2.$$

Proof: When $m = n = 2$, the expression in Lemma 5 can be written as

$$\sum_{i \in I(1)} d_{i1} + \sum_{i \in I(2)} d_{i2} + x_1^2 + x_2^2 \leq \sum_{i \in I^*(1)} d_{i1} + \sum_{i \in I^*(2)} d_{i2} + x_1 x_1^* + x_1 * + x_2 x_2^* + x_2 * .$$

Substituting $x_2 = n - x_1$ and $x_2^* = n - x_1^*$ into the expression on the right, we obtain

$$\sum_{i \in I^*(1)} d_{i1} + \sum_{i \in I^*(2)} d_{i2} + x_1 x_1^* + x_1 * + (n - x_1)(n - x_1^*) + (n - x_1^*)$$

$$= \sum_{i \in I^*(1)} d_{i1} + \sum_{i \in I^*(2)} d_{i2} + n^2 - n x_1^* - n x_1 + n + 2 x_1 x_1^*.$$

Using the result of Lemma 5, if the following relationship holds then we will immediately obtain the desired price of anarchy result:

$$\sum_{i \in I^*(1)} d_{i1} + \sum_{i \in I^*(2)} d_{i2} + x_1 x_1^* + x_1 * + x_2 x_2^* + x_2 * \leq 2 \cdot \left(\sum_{i \in I^*(1)} d_{i1} + \sum_{i \in I^*(2)} d_{i2} + x_1^{*2} + x_2^{*2} \right).$$

The expression on the right is twice the central optimal cost, and this simplifies to

$$2 \cdot \left(\sum_{i \in I^*(1)} d_{i1} + \sum_{i \in I^*(2)} d_{i2} + x_1^{*2} + (n - x_1^*)^2 \right).$$

We wish to know, then, for which values of x_1 and x_1^* the following is true:

$$\sum_{i \in I^*(1)} d_{i1} + \sum_{i \in I^*(2)} d_{i2} + n^2 - n x_1^* - n x_1 + n + 2 x_1 x_1^* \leq 2 \cdot \left(\sum_{i \in I^*(1)} d_{i1} + \sum_{i \in I^*(2)} d_{i2} + x_1^{*2} + (n - x_1^*)^2 \right).$$

After algebraic manipulation, this is equivalent to

$$0 \leq \sum_{i \in I^*(1)} d_{i1} + \sum_{i \in I^*(2)} d_{i2} + 4 x_1^{*2} + n^2 + n x_1 - 3 n x_1^* - 2 x_1 x_1^* - n.$$

Since $n = 2$, x_1 and x_1^* can each take on the values 0, 1, or 2. Enumerating all nine cases shows that the inequality holds for all possible values, and again the network in Figure 2 shows the bound is tight. Thus

$$\frac{z(\alpha = 1)}{z^*} = 2$$

when $n = m = 2$. □

We have shown for several special cases that the price of anarchy when $\alpha_j = 1$ for all j is 2. We further conjecture that this is the worst-case bound on the performance of decentralized system in this case for general network structures.

B.1.2 When Congestion Weights Equal 0 for All Facilities

We first examine the case in which the customers are in a single location and all facilities are equidistant from that location. Suppose we have chosen to locate m facilities and that the travel time from the customer location A to each facility is represented by d. We wish develop closed-form expressions for the cost of a centralized solution in which a central planner assigns customers to the facilities. The cost of a solution is again given by the sum of the total travel time and the total congestion experienced by all customers in the system. The objective of the central planner is to minimize this cost, and the minimum cost can be expressed as follows.

Theorem 12 *The cost of the optimal assignment of n customers, all of whom are located at A, to m facilities all with travel time d from A, is given by*

$$C_m^* = nd + \frac{1}{m}n^2.$$

Proof: We assume that n is divisible by m. Let k_j be the number of customers assigned to facility j by the central planner. From the definition of the total system cost, we know that

$$C_m = \sum_{j=1}^{m}(k_j d + k_j^2).$$

We wish to find the values of k_j that minimize C_m. First, observe that since the total number of customers is n, then $k_m = n - \sum_{j=1}^{m-1} k_j$. Substituting, we have

$$C_m = \sum_{j=1}^{m-1}(k_j d + k_j^2) + (n - \sum_{j=1}^{m-1} k_j)d + (n - \sum_{j=1}^{m-1} k_j)^2.$$

Examining the rate of change of C_m with respect to the change in each k_j, we have

$$\frac{\Delta C_m}{\Delta k_j} = 4k_j + d - d - 2n + 2 \sum_{l \neq j, l < m} k_l$$

for $j = 1, \ldots, m-1$.

We propose that k^*, the vector of assignments that minimizes the total system cost, is given by $k_j^* = \frac{1}{m}n$ for all j. We need to show that k_m^* has the same form as that of k_j^* for $j = 1, \ldots, m-1$, that the proposed k^* satisfies the system when the rates of change are equated to zero, and that this solution yields a global minimum of the cost function.

To obtain k_m^*, substitute the proposed form of k_j^*'s:

$$k_m^* = n - \sum_{j=1}^{m-1} k_j^* = n - \sum_{j=1}^{m-1} (\frac{1}{m}n) = \frac{m-1}{m}n = \frac{1}{m}n.$$

Thus, the form of k_m^* is identical to that of the other k_j^*'s.

Substituting $k_j^* = \frac{1}{m}n$ for k_j in the rate of change expressions yields

$$4k_j - 2n + 2\sum_{l\neq j, l<m} k_l = 4(\frac{1}{m}n) - 2n + 2(m-2)\frac{1}{m}n = \frac{4}{m}n - 2n + 2n - \frac{4}{m}n = 0,$$

thus indicating that this is a local minima.

Finally, we observe that when k values are permitted to be non-integer, the Hessian (matrix of second derivatives) of the continuous form of the cost function is positive definite at every point. Therefore, the continuous form of the cost function is convex, which implies that a local minimum is also the global minimum. The vector k^* is a local minimum of the continuous form of the cost function, and since it is also integer-valued, this assignment of customers to locations is indeed a global minimum of the true (integer-valued) cost function. We write $C_m(k^*) = C_m^*$.

All that remains is substituting the k_j^* values into the original expression for total system cost. We have

$$C_m^* = \sum_{j=1} m(\frac{1}{m}nd + (\frac{1}{m}n)^2) = m(\frac{1}{m}nd + \frac{1}{m^2}n^2) = nd + \frac{1}{m}n^2,$$

as proposed. □

In this expression, the first term captures the travel time from the customer location to facilities, while the second gives the congestion at the facilities. If all facilities are equidistant from the customers, the assignment that minimizes congestion is that which splits customers equally among facilities.

We can also characterize the ratio between a centralized solution and a decentralized one in the case where customers choose based only on travel times.

Corollary 5 *In the case in which all facilities have equal travel time d from customer location A and $\alpha_j = 0$ for all j, the price of anarchy is $O(m)$.*

Proof: Suppose that all facilities have travel time d from customer location A, except for facility m, whose travel time is $d-\epsilon$. (Equivalently, imagine that all customers make the same choice of facility, leading to the worst possible congestion.) Then in the decentralized scenario all customers will choose facility m, and the realized cost of these choices is $n(d-\epsilon)+n^2$. Letting ϵ go to 0, the decentralized cost is $nd+n^2$. Thus, the ratio between the decentralized realization and the centralized optimal cost is

$$\text{Price of Anarchy} = \frac{(nd+n^2)}{(nd+\frac{1}{m}n^2)} = O(m).\Box$$

This result is intuitive, for the worst congestion occurs when all customers choose the same facility.

These ideas can also be extended to the case where the travel time, d_{Aj}, from the common customer location A to facility j is general. We consider the same problem of a centralized planner determining the assignment of customers to facilities to minimize the total travel time plus congestion, using the latency functions already described. Under this scenario, the optimal central cost can be expressed in closed form.

Theorem 13 *The cost of the optimal assignment of n customers, all of whom are located at A, to m facilities $j=1,\ldots,m$ with travel times d_{Aj} from A, the minimum cost is given by*

$$C^* = \frac{1}{m}n^2 - \frac{m-1}{4m}\sum_{j=1}^{m} d_{Aj}^2 + \frac{1}{m}n\sum_{j=1}^{m} d_{Aj} + \frac{1}{2m}\sum_{j=1}^{m}\sum_{k>j} d_{Aj}d_{Ak}.$$

Proof: Assume n is divisible by m. Let k_j be the number of customers assigned to facility j. Then the cost of an assignment is

$$C = \sum_{j=1}^{m}(k_j d_{Aj} + k_j^2).$$

Since the total number of customers must be n, we have $k_m = n - \sum_{j=1}^{m-1} k_j$. Thus,

$$C = \sum_{j=1}^{m-1} k_j d_{Aj} + d_{Am}(n - \sum_{i=1}^{m-1} k_j) + \sum_{j=1}^{m-1} k_j^2 + (n - \sum_{j=1}^{m-1} k_j)^2.$$

Examining the rate of change of the cost function with respect to changes in each k_j, we have

$$\frac{\Delta C}{\Delta k_j} = 4k_j + d_{Aj} - d_{Am} - 2n + 2\sum_{l \neq j, l<m} k_l$$

for $j = 1, \ldots, m-1$.

We propose that k^*, the vector of assignments that minimizes total system cost, is given by

$$k_j^* = \frac{1}{m}n - \frac{m-1}{2m}d_{Aj} + \frac{1}{2m}\sum_{l \neq j} d_{Al} \quad \forall\ j.$$

We must show that k_m^* has the same form as k_j^* for $j = 1, \ldots, m-1$, that the proposed k^* satisfies the system when the rates of change are equated to zero, and that this solution yields a global minimum of the cost function. First, we substitute for k_m^*:

$$\begin{aligned}
k_m^* = n - \sum_{j=1}^{m-1} k_j^* &= n - \sum_{j=1}^{m-1}\left(\frac{1}{m}n - \frac{m-1}{2m}d_{Aj} + \frac{1}{2m}\sum_{l \neq j} d_{Al}\right) \\
&= n - \frac{m-1}{m}n + \frac{m-1}{2m}\sum_{j=1}^{m-1} d_{Aj} - \frac{1}{2m}\sum_{j=1}^{m-1}\sum_{l \neq j} d_{Al} \\
&= \frac{1}{m}n + \frac{m-1}{2m}\sum_{l \neq m} d_{Al} - \frac{m-2}{2m}\sum_{l \neq m} d_{Al} - \frac{m-1}{2m}d_{Am} \\
&= \frac{1}{m}n - \frac{m-1}{2m}d_{Am} + \frac{1}{2m}\sum_{l \neq m} d_{Al}.
\end{aligned}$$

Thus, the form of k_m^* is the same as that of the other k_j^*'s.

Now, substituting $k_j^* = \frac{1}{m}n - \frac{m-1}{2m}d_{Aj} + \frac{1}{2m}\sum_{l \neq j} d_{Al}$ for k_j in the rate of change expressions yields:

$$\begin{aligned}
\frac{\Delta C}{\Delta k_j^*} &= 4\left(\frac{1}{m}n - \frac{m-1}{2m}d_{Aj} + \frac{1}{2m}\sum_{l \neq j} d_{Al}\right) + d_{Aj} - d_{Am} - 2n + 2\sum_{l \neq j, l<m}\left(\frac{1}{m}n - \frac{m-1}{2m}d_{Al} + \frac{1}{2m}\sum_{k \neq l} d_{Ak}\right) \\
&= \frac{4}{m}n - \frac{4(m-1)}{2m}d_{Aj} + \frac{4}{2m}\sum_{l \neq j} d_{Al} + d_{Aj} - d_{Am} - 2n \\
&\quad + 2\sum_{l \neq j, l<m}\frac{1}{m}n - \frac{2(m-1)}{2m}\sum_{l \neq j, l<m} d_{Al} + \frac{2}{2m}\sum_{l \neq j, l<m}\sum_{k \neq l} d_{Ak}.
\end{aligned}$$

We will now show that, by simplification and grouping, the rate of change expression reduces to 0 as desired.

1. The last term,
$$\frac{2}{2m}\sum_{l\neq j, l<m}\sum_{k\neq l} d_{Ak},$$
simplifies to
$$\frac{2}{2m}[(m-2)d_{Aj} + (m-3)\sum_{l\neq j, l<m} d_{Al} + (m-2)d_{Am}].$$

2. Grouping the terms involving n gives
$$\frac{4}{m} + \frac{2(m-2)}{m} - 2 = \frac{2m}{m} - 2 = 0.$$

3. Grouping the terms involving d_{Aj} gives
$$1 - \frac{4(m-1)}{2m} + \frac{2(m-2)}{2m} = \frac{2m - 4m + 4 + 2m - 4}{2m} = 0.$$

4. Grouping the terms involving $\sum_{l\neq j, l\neq m} d_{Al}$ gives
$$\frac{4}{2m} - \frac{2(m-1)}{2m} + \frac{2(m-3)}{2m} = \frac{4 - 2m + 2 + 2m - 6}{2m} = 0.$$

5. Grouping the terms involving d_{Am} gives
$$\frac{4}{2m} - 1 + \frac{2(m-2)}{2m} = \frac{4 - 2m + 2m - 4}{2m} = 0.$$

Thus, the proposed k_j^*'s satisfy the equations in which the rates of change are set to 0. Finally, the vector k^* is the global minimum using the same argument as in Theorem 12, namely that the Hessian (matrix of second derivatives) of the continuous form of the cost function is positive definite, k^* is the global minimum of the continuous form of the cost function, and since it is integer-valued, it is also the global minimum of the true cost function.

All that remains is to substitute the optimal k_j^* values into the cost function, $C^* = \sum_{j=1}^{m}(k_j^* d_{Aj} + k_j^{*2})$. The following steps outline this substitution.

1. Consider only the first term in the parentheses for a single j and substitute for k_j^*:

$$k_j^* d_{Aj} = \frac{1}{m} n d_{Aj} - \frac{m-1}{2m} d_{Aj}^2 + \frac{1}{2m} d_{Aj} \sum_{l \neq j} d_{Al}.$$

2. Consider only the second term in the parentheses for a single j and substitute for k_j^*:

$$k_j^{*2} = \frac{1}{m^2} n^2 - \frac{2(m-1)}{2m^2} n d_{Aj} + \frac{2}{2m^2} n \sum_{l \neq j} d_{Al} + \frac{(m-1)^2}{4m^2} d_{Aj^2} -$$

$$\frac{2(m-1)}{4m^2} d_{Aj} \sum_{l \neq j} d_{Al} + \frac{1}{4m^2} (\sum_{l \neq j} d_{Al})^2$$

$$= \frac{1}{m^2} n^2 - \frac{m-1}{m^2} n d_{Aj} + \frac{1}{m^2} n \sum_{l \neq j} d_{Al} + \frac{(m-1)^2}{4m^2} d_{Aj}^2 -$$

$$\frac{m-1}{2m^2} d_{Aj} \sum_{l \neq j} d_{Al} + \frac{1}{4m^2} \sum_{l \neq j} d_{Al}^2 + \frac{2}{4m^2} \sum_{l \neq j} \sum_{k > l} d_{Al} d_{Ak}.$$

3. Combine the terms from Steps 1 and 2 for a single j:

$$k_j^* d_{Aj} + k_j^{*2} =$$

$$\frac{1}{m^2} n^2 + (\frac{1}{m} - \frac{m-1}{m^2}) n d_{Aj} + \frac{1}{m^2} n \sum_{l \neq j} d_{Al} + (\frac{(m-1)^2}{4m^2} - \frac{m-1}{2m}) d_{Aj}^2$$

$$+(\frac{1}{2m} - \frac{m-1}{2m^2}) d_{Aj} \sum_{l \neq j} d_{Al} + \frac{1}{4m^2} \sum_{l \neq j} d_{Al}^2 + \frac{1}{2m^2} \sum_{l \neq j} \sum_{k > l} d_{Al} d_{Ak}.$$

4. Simplify terms:

   - Combine terms involving nd_{Aj} to obtain

$$\frac{1}{m} - \frac{m-1}{m^2} = \frac{m - m + 1}{m^2} = \frac{1}{m^2}.$$

   - Combine terms involving d_{Aj}^2 to obtain

$$\frac{(m-1)^2}{4m^2} - \frac{m-1}{2m} = \frac{m^2 - 2m + 1 - 2m^2 + 2m}{4m^2} = \frac{-(m^2-1)}{2m^2} = \frac{-(m+1)(m-1)}{4m^2}.$$

- Combine terms involving $d_{Aj}\sum_{l\neq j} d_{Al}$ to obtain

$$\frac{1}{2m} - \frac{m-1}{2m^2} = \frac{m-m+1}{2m^2} = \frac{1}{2m^2}.$$

5. Rewrite based on simplification:

$$k_j^* d_{Aj} + k_j^{*2} =$$

$$\frac{1}{m^2}n^2 + \frac{1}{m^2}n\sum_{l=1}^{m} d_{Al} - \frac{(m+1)(m-1)}{4m^2}d_{Aj}^2$$

$$+\frac{1}{2m^2}d_{Aj}\sum_{l\neq j} d_{Al} + \frac{1}{4m^2}\sum_{l\neq j} d_{Al}^2 + \frac{1}{2m^2}\sum_{l\neq j}\sum_{k>l} d_{Al}d_{Ak}.$$

6. Sum over all j:

$$\sum_{j=1}^{m}(k_j^* d_{Aj} + k_j^{*2}) =$$

$$\frac{1}{m}n^2 + \frac{1}{m}n\sum_{j=1}^{m} d_{Aj} - \frac{(m+1)(m-1)}{4m^2}\sum_{j=1}^{m} d_{Aj}^2$$

$$+\frac{1}{2m^2}\sum_{j=1}^{m} d_{Aj}(\sum_{l\neq j} d_{Al}) + \frac{m-1}{4m^2}\sum_{j=1}^{m} d_{Aj}^2 + \frac{m-2}{2m^2}\sum_{j=1}^{m}\sum_{l>j} d_{Aj}d_{Al}.$$

7. Simplify terms:

- Rewrite to obtain

$$\sum_{j=1}^{m} d_{Aj}(\sum_{l\neq j} d_{Al}) = 2\sum_{j=1}^{m}\sum_{l>j} d_{Aj}d_{Al}.$$

- Combine terms involving $\sum_{j=1}^{m}\sum_{l>j} d_{Aj}d_{Al}$ to obtain

$$\frac{2}{2m^2} + \frac{m-2}{2m^2} = \frac{m}{2m^2} = \frac{1}{2m}.$$

- Combine terms involving $\sum_{j=1}^{m} d_{Aj}^2$ to obtain

$$\frac{m-1}{4m^2} - \frac{(m+1)(m-1)}{4m^2} = \frac{m-1-m^2+1}{4m^2} = \frac{-m^2+m}{4m^2} = \frac{-m+1}{4m} = -\frac{m-1}{4m}.$$

8. Rewrite:

$$C_m^* = \sum_{j=1}^{m}(k_j^* d_{Aj} + k_j^{*2}) =$$

$$\frac{1}{m}n^2 - \frac{m-1}{4m}\sum_{j=1}^{m} d_{Aj}^2 + \frac{1}{m}n\sum_{j=1}^{m} d_{Aj} + \frac{1}{2m}\sum_{j=1}^{m}\sum_{l>j} d_{Aj}d_{Al}.$$

Thus, the optimal cost for the centralized problem is given by the equation presented. □

Again, the first term comes from the facility congestion. However, the additional terms now capture the trade-off between facility congestion and travel time. The optimal assignment in the case with general travel times can be seen as an adjustment to that in the case with equal travel times. If we fix all travel times except d_{Aj}, we see that as d_{Aj} increases it is less attractive to assign customers to facility j. Travel time begins to have more influence in the assignment decision than congestion. But holding d_{Aj} fixed and varying the travel times to other facilities, we see that if other facilities require long travel times we would like to assign some additional customers to facility j. As before, we can characterize the ratio of the optimal cost and the cost of a decentralized solution in the case with $\alpha_j = 0$ for all j.

Corollary 6 *Suppose travel times to facility j from customer location A, denoted by d_{Aj}, are sorted in non-decreasing order such that $d_{A1} < d_{A2} < \ldots < d_{Am}$ and that $\alpha_j = 0$ for all j. Then the price of anarchy is $O(m)$.*

Proof: The realized cost of the decentralized choices is $nd_{A1} + n^2$, since all customers choose the closest facility. Using the results of Theorem 13 on the form of the centralized optimal cost, the ratio is given by

$$\frac{nd_{A1} + n^2}{\frac{1}{m}n^2 - \frac{m-1}{4m}\sum_{j=1}^{m} d_{Aj}^2 + \frac{1}{m}n\sum_{j=1}^{m} d_{Aj} + \frac{1}{2m}\sum_{j=1}^{m}\sum_{l>j} d_{Aj}d_{Al}}$$

$$= \frac{m(nd_{A1} + n^2)}{n^2 - \frac{m-1}{4}\sum_{j=1}^{m} d_{Aj}^2 + n\sum_{j=1}^{m} d_{Aj} + \frac{1}{2}\sum_{j=1}^{m}\sum_{l>j} d_{Aj}d_{Al}}$$

$$= O(m).\square$$

Again, this ratio is dominated by the number of customers but now the centralized solution also has terms to account for the trade-off in travel time and congestion. We note that the strict sorting of travel times is used here for a clear exposition but is not necessary. In fact, a technique identical to that used in the proof of Corollary 5 in which a single facility has travel time at least ϵ less than any other facility can be used to relax the supposition.

B.2 Computing the Price of Anarchy

In Section 2.5.3, we proved that the price of anarchy in the case where $\alpha_j = 1$ for all j can be at least 2. Here we present a computational approach we developed for determining the price of anarchy for any given instance of the Facility-Specific Congestion Weights Problem when all α's are equal to 1. This approach can also be used to study the average price of anarchy computationally for different network structures.

B.2.1 Optimization Model

To calculate the price of anarchy for a given instance of the fixed location problem, we must find both the optimal solution and the Nash equilibrium solution with the highest cost. The optimal solution is obtained by solving the planner's problem presented in Section 2.2. One approach for finding the worst Nash equilibrium is by solving the following optimization problem, where binary decision variable x_{ij} is 1 if customer i is served at facility j and 0 otherwise:

$$\text{Maximize} \quad \sum_{i=1}^{n}\sum_{j=1}^{m} d_{ij}x_{ij} + \sum_{j=1}^{m}\left(\sum_{i=1}^{n} x_{ij}\right)^2 \tag{84}$$

$$\sum_{j=1}^{m} x_{ij} = 1 \qquad \forall i \tag{85}$$

$$(d_{ij} + \sum_{l=1}^{n} x_{lj}) \cdot x_{ij} \leq d_{ik} + \sum_{l=1}^{n} x_{lk} + 1 \ \ \forall\ i,\ k \neq j \tag{86}$$

$$x_{ij} \in \mathbb{B} \qquad\qquad \forall i, j \tag{87}$$

Constraint (85) ensures that each customer is served at exactly one facility. The Equilibrium Condition of each individual with $\alpha_j = 1$ for all j (Inequality (1) from Section 2.2) is enforced by Constraint (86), while Constraint (87) indicates the binary decision variables. The objective function (84) in this representation is the same as that in the central planner's problem. However, we are now maximizing that value to identify the equilibrium solution with greatest cost. By changing the objective sense to minimization, we are also able to use this model to identify the best equilibrium solution. This model is an important contribution since most research on network games seeks to identify a single equilibrium solution, rather than the best or worst.

While this formulation is follows naturally from the system objective and the Equilibrium Condition, both the objective function and Constraint (86) are nonlinear. The constraint can be made linear by introducing a large number M and rewriting as follows:

$$d_{ij} + \sum_{l=1}^{n} x_{lj} \leq d_{ik} + \sum_{l=1}^{n} x_{lk} + 1 + M(1 - x_{ij}) \ \ \forall\ i,\ k \neq j$$

The objective function can also be converted to an equivalent linear expression using the following procedure.

- Rewrite the objective function in matrix form as $x'Qx + Dx$, where D is the matrix of travel times from customers to facilities and x' and Q are given by

$$x' = (x_{11} \ldots x_{k1} \ldots x_{n1}\ x_{12} \ldots x_{k2} \ldots x_{n2} \ldots x_{nm}),$$

$$Q = \begin{pmatrix} \mathbf{1} & \ldots & \mathbf{1} & \ldots & \mathbf{1} & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 \\ \mathbf{1} & \ldots & \mathbf{1} & \ldots & \mathbf{1} & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 \\ \mathbf{1} & \ldots & \mathbf{1} & \ldots & \mathbf{1} & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 \\ 0 & \ldots & 0 & \ldots & 0 & \mathbf{1} & \ldots & \mathbf{1} & \ldots & \mathbf{1} & \ldots & 0 \\ 0 & \ldots & 0 & \ldots & 0 & \mathbf{1} & \ldots & \mathbf{1} & \ldots & \mathbf{1} & \ldots & 0 \\ 0 & \ldots & 0 & \ldots & 0 & \mathbf{1} & \ldots & \mathbf{1} & \ldots & \mathbf{1} & \ldots & 0 \\ \ldots & & & & & & & & & & & \\ 0 & \ldots & 0 & \ldots & 0 & 0 & \ldots & 0 & \ldots & 0 & \ldots & \mathbf{1} \end{pmatrix}.$$

- Rewrite the objective function as $x'Qx + Dx = x'(Q - \lambda I)x + \lambda x'x + Dx$, where λ is the largest eigenvalue of Q and I is the identity matrix.

- To find the largest eigenvalue, recall from linear algebra that for any square matrix the number of nonzero eigenvalues is equal to the matrix rank. Clearly the rank of Q is m. Moreover, the sum of the diagonal entries of a matrix is equal to the sum of its eigenvalues. Therefore, for Q we have that the sum of the eigenvalues must be nm. This implies that all nonzero eigenvalues of Q are equal to n; therefore $\lambda = n$.

- Rewrite the objective function as $x'(Q - nI)x + nx'x + Dx = x'(Q - nI)x + nx + Dx$, where the second expression follows from the fact that all $x_{ij's}$ are binary.

- Rewrite the objective function as a summation, leading to

$$\sum_i \sum_j (1-n)x_{ij}^2 + \sum_j \sum_i \sum_{k \neq i} x_{ij}x_{kj} + \sum_i \sum_j (nx_{ij} + d_{ij}x_{ij}).$$

- Note first that $x_{ij}^2 = x_{ij}$ since the variables are binary. In addition, observe that $x_{ij}x_{kj} = 1 \iff x_{ij} = 1 \; and \; x_{kj} = 1$. Using these two facts, rewrite the objective function as

$$\sum_i \sum_j (1-n)x_{ij} + \sum_j \sum_i \sum_{k \neq i} y_{ikj} + \sum_i \sum_j (nx_{ij} + d_{ij}x_{ij}),$$

where y_{ikj} is a binary variable equal to 1 if both x_{ij} and x_{kj} are 1 and 0 otherwise. To enforce this relationship, add the constraints $y_{ikj} \leq x_{ij}$ and $y_{ikj} \leq x_{kj}$.

- Finally, simplify the objective function to obtain

$$\sum_i \sum_j x_{ij} + \sum_j \sum_i \sum_{k \neq i} y_{ikj} + \sum_i \sum_j d_{ij}x_{ij}.$$

Making use of the reformulated objective function and additional constraints, the problem of finding the worst Nash equilibrium becomes:

$$\text{Maximize} \quad \sum_i \sum_j x_{ij} + \sum_j \sum_i \sum_{k \neq i} y_{ikj} + \sum_i \sum_j d_{ij}x_{ij} \tag{88}$$

$$\sum_{j=1}^{m} x_{ij} = 1 \qquad \forall i \tag{89}$$

$$d_{ij} + \sum_{l=1}^{n} x_{lj} \leq d_{ik} + \sum_{l=1}^{n} x_{lk} + 1 + M(1 - x_{ij}) \quad \forall\ i,\ k \neq j \tag{90}$$

$$y_{ikj} \leq x_{ij} \qquad \forall i, j, k \neq i \tag{91}$$

$$y_{ikj} \leq x_{kj} \qquad \forall i, j, k \neq i \tag{92}$$

$$x_{ij} \in \mathbb{B} \qquad \forall i, j \tag{93}$$

This formulation provides the main component of a solution procedure used to compute the price of anarchy for a given instance of the FSCWP when $\alpha_j = 1$ for all j.

B.2.2 Solution Procedure

Using the integer programming formulation just developed, we implement the following procedure to study the price of anarchy.

1. Solve the planner's problem to determine the central optimal solution.

2. Solve the integer program to identify the Nash equilibrium solution with the highest cost.

   (a) Initialize with the Nash equilibrium solution found using the minimum cost flow procedure described in Theorem 1.

   (b) Preprocess to eliminate variables if possible.

   (c) Add a constraint requiring that the objective function value be less than 2.5 times the optimal solution, as known from the literature [8].

   (d) Solve the integer program and report the cost of the worst Nash equilibrium.

3. Calculate the price of anarchy.

Steps 2a, 2b, and 2c were added to this solution procedure to reduce solution times for the integer programming problem. In a small test study, the optimization software did not find a feasible integer solution for many cases. Using knowledge about the minimum cost flow transformation from Theorem 1 enabled the identification of such a solution very quickly. Some variables can be preprocessed out of the formulation based on the instance data. If a facility is located such that the travel time is greater than $d_{ij_{min}} + n - 1$ from customer i, where j_{min} is i's closest facility, then i will never choose that facility. The associated assignment variable can be eliminated in this case. Finally, the literature provides a known upper bound of 2.5 on the price of anarchy. Further testing is needed to determine whether its inclusion can speed the computation times. Studies using this approach on different kinds of network instances may help provide insight about general relationships between the number of customers, the number of facilities, and the price of anarchy. They may also lead to additional ideas for proving the conjecture about the price of anarchy for general networks.

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